Graphs of curves & arcs quasi-isometric to big mapping class groups

Anschel Schaffer-Cohen
(University of Pennsylvania/Temple University)

Outline:

- Define
- Results
- Methods

A big mapping class group is the mapping class group of an infinite-type surface.

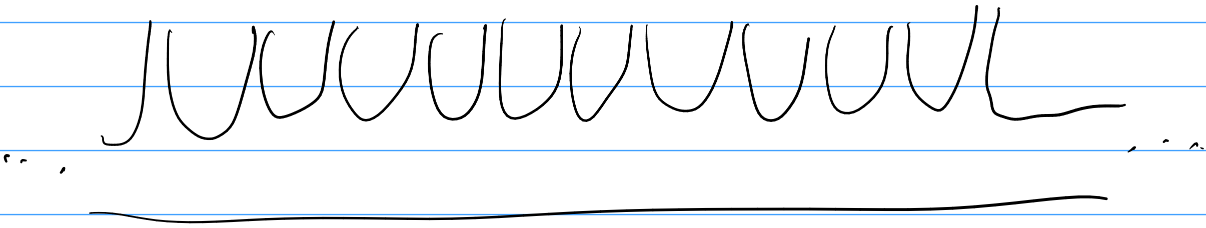
Connected, orientable, w/o bdry

If a surface is compact or compact w/ fin. many punctures,

we say it has "finite type" (f.t.)
Otherwise "infinite type"

f.t. $\Leftrightarrow$ $\pi_1(\Sigma)$ is f.g.

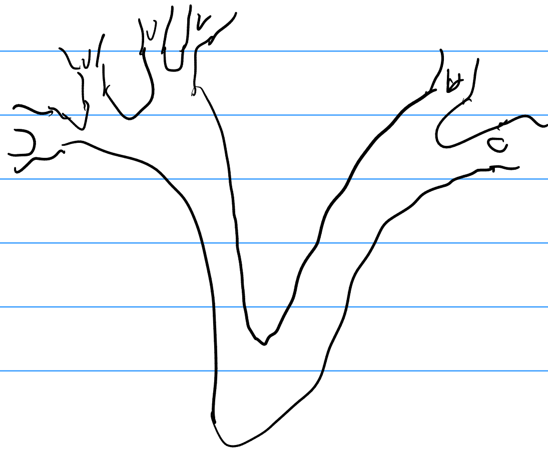
"Jacob's ladder"



"flute"
bi-infinite

"shark tank"

"cantor tree"



The Mapping class group $MCG(\Sigma)$ is the group of orientation-preserving homeomorphisms of Σ up to isotopy

$$\mathrm{Homeo}^+(\Sigma)/\mathrm{Homeo}^0(\Sigma) = \pi_0(\mathrm{Homeo}^+(\Sigma))$$

How are classical and big MCG different?

finitely generated	uncountable
curve graph δ-hyp & inf-diam	curve graph diam 2
hyperbolic structures all related	no
discrete	there is some topology

How do we think about quasi-isometry?

Big idea (Rosendal): Many concepts and applications from GGT apply to topological gps if we replace "finite" w/ "coarsely bdd"

<u>Defn</u> A subset A of a top. gp G is CB if it is bdd wrt every left-invariant pseudo-metric on G that

is compatible w/ its topology.

equiv Defn $A \subseteq G$ is CB if whenever G acts continuously on a metric space X, $\forall x_0 \in X$, $Ax_0 = \{ax_0 \mid a \in A\}$ is bdd.

Instead of studying discrete, f.g. groups, we study locally CB, CB-generated groups.

Payoff (Rosendal): If A & B are two CB gen sets for such a group, their word metrics are QI.

Mann-Rafi classified which surfaces have locally CB, CB-generated MCG

Question: Is there a nice graph w/ a MCG-action induces a quasi-isometry via orbit map?

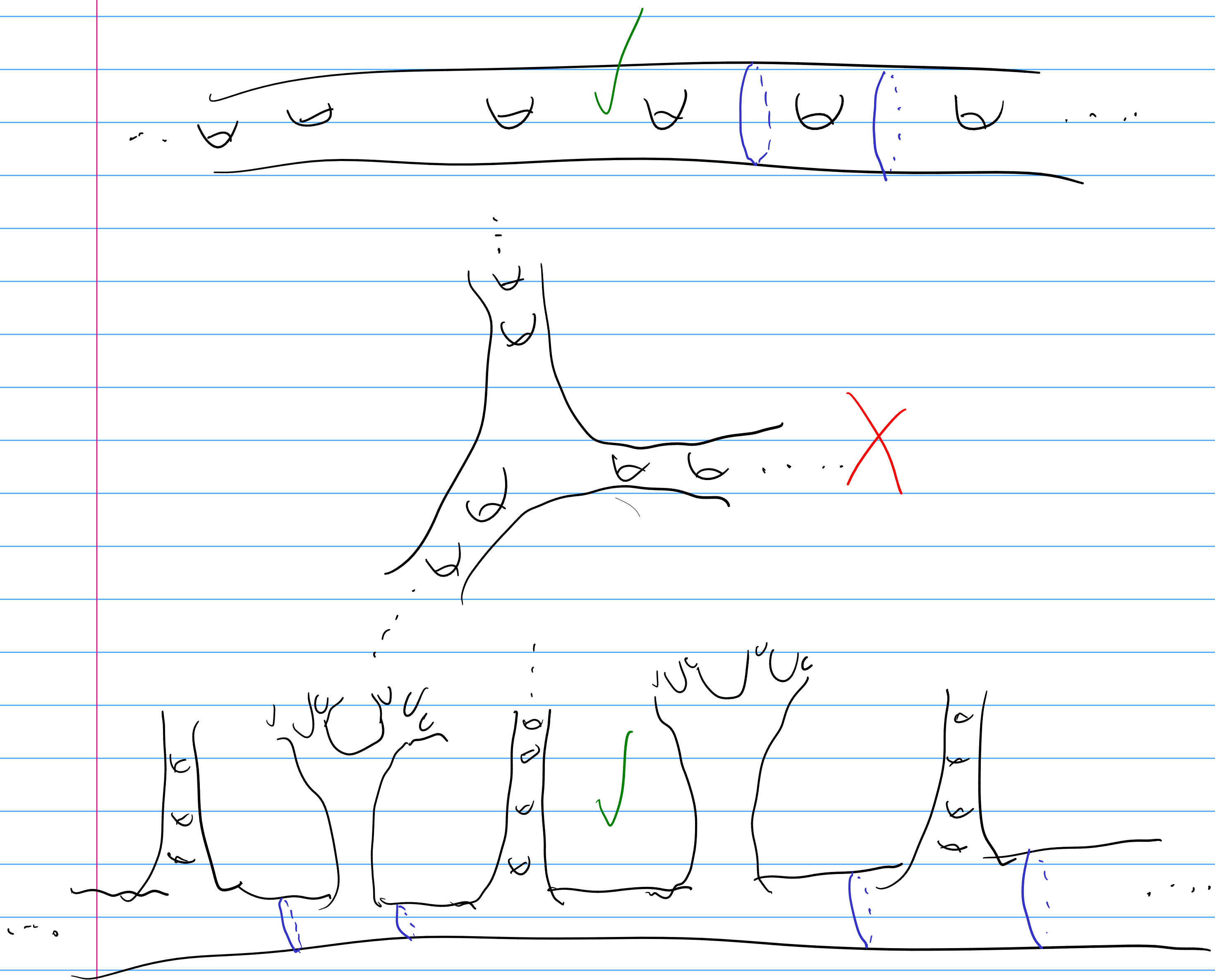
Graphs of curves

vertices are isotopy classes of SCC

Graphs of arcs

vertices are isotopy classes of simple paths connecting punctures

A surface is translatable if it admits a "translation"



The translatable curve graph $TCG(\Sigma)$ is the graph whose vertices are curves separating the two special ends and with two curves connected by an edge if they are disjoint and cobound a subsurface homeo to one of a finite set of surfaces we define.

In fact $MCG(\Sigma)$ is quasi-isometric to $TCG(\Sigma)$

Thm: Suppose Σ is an inf-type surface w/ tame end space, $MCG(\Sigma)$ is CB-generated, but not CB, then TFAE

1) There is some graph of curves on which the action of MCG induces a QI.
2) Σ is translatable
3) Σ has no nondisplaceable finite-type subsurfaces

(Horbez-Qing-Rafi: "avenue surfaces")

If $\varphi: MCG \xrightarrow{qi} \Gamma$ the preimage of a vertex must be CB

$\Sigma = \mathbb{R}^2 \setminus$ Cantor set, this is well-studied

Bavard: introduced the loop graph, vertices are arcs, edges are disjoint reps

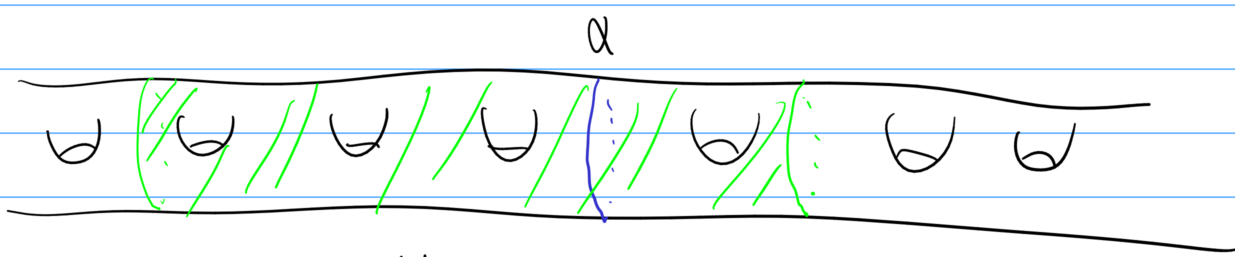
Bavard: this graph is δ-hyperbolic and ∞-diam

Bavard-Walker: described its bdry

Thm: The action of $MCG(\Sigma)$ on the loop graph induces a QI.

Cor: $MCG(\mathbb{R}^2 \setminus \text{cantor})$ is δ-hyperbolic.

One important tool: (Rosendal) $A \subseteq G$ is CB iff $\forall$ identity nbhd $V \subseteq G$, $\exists$ a finite $F \subseteq G$, $k \in \mathbb{N}$ s.t. $A \subseteq (FV)^k$



WTS stab(α) is CB

V is some id nbhd, fixes S

s'pose $f \in MCG$ $f(\alpha) = \alpha$; translate enough, α is on one side of the translate of S