

UNIVERSITY OF TORONTO
DEPARTMENT OF MATHEMATICS
MAT 235 Y – CALCULUS II
FALL-WINTER 2007-08
ASSIGNMENT #1. DUE ON OCTOBER 4.
PROBLEMS

DO NOT SUBMIT YOUR SOLUTIONS WITHOUT THE COVER PAGE.
READ THE INSTRUCTIONS WRITTEN ON THAT PAGE.

1. A brief review of single variable calculus.

a) Evaluate $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{x^2 + 8 \sin x}{1 - \cos(4x)} \right)$, if the limit exists.

b) Find the absolute maximum and the absolute minimum, if any, of the function $f(x) = e^{6|x-1|-x^2}$.

2. More review on single variable calculus.

a) Evaluate the improper integral $\int_{-1}^{\infty} \left(\frac{x^4}{1+x^6} \right)^2 dx$

b) Let W be the region consisting of all the points (x, y) such that $1 \leq x \leq e$ and $\ln x \leq y \leq x$.

Compute the volume of the solid generated when the region W is rotated about the line $x = -1$.

3. Calculus with parametric curves.

Consider the curve C with parametric equations $x = 2 \int_1^t \sqrt{2+u^2} du$ and $y = t + \frac{1}{3}t^3$.

a) Find the slope of the tangent line to the curve C at the point where $t = 2$.

b) Find the values of t corresponding to each of the inflection points, if any, of the curve C .

c) Find the area of the surface obtained when the portion of C with $0 \leq t \leq \sqrt{2}$ is rotated about the x -axis.

4. Polar coordinates.

a) Find the coordinates of each of the points of intersection of the polar curves $r = 1 - \sin \theta$ and $r = 1 + 3 \sin \theta$.

b) Compute the area of the region enclosed by the polar curve $r = \min(1 - \cos \theta, 2 + 2 \sin \theta)$, where $0 \leq \theta \leq 3\pi/2$.

c) Compute the length of the polar curve $r = \max(1 - \cos \theta, 2 + 2 \sin \theta)$, where $0 \leq \theta \leq 3\pi/2$.

5. Three dimensional coordinate systems. Vectors.

a) Find an equation of the largest sphere that passes through the point $(-1, 1, 4)$ and is such that each of its points (x, y, z) satisfies the condition $x^2 + y^2 + z^2 \leq 136 + 2(x + 2y + 3z)$.

b) Let \mathbf{v}_1 and \mathbf{v}_2 be any pair of three-dimensional vectors such that $|\mathbf{v}_1| = 2$, $|\mathbf{v}_2| = 3$ and $\mathbf{v}_1 \cdot \mathbf{v}_2 = 5$.

Let $\mathbf{v}_3 = \text{proj}_{\mathbf{v}_1} \mathbf{v}_2$, $\mathbf{v}_4 = \text{proj}_{\mathbf{v}_2} \mathbf{v}_3$, $\mathbf{v}_5 = \text{proj}_{\mathbf{v}_3} \mathbf{v}_4$, and so on. Compute $\sum_{i=1}^{\infty} |\mathbf{v}_i|$.

c) Let L_1 be the line that passes through the points $(1, 2, 6)$ and $(2, 4, 8)$ and let L_2 be the line of intersection of the planes Π_1 and Π_2 , where Π_1 is the plane with equation $x - y + 2z = -1$ and Π_2 is the plane that passes through the points $(3, 2, -1)$, $(0, 0, 1)$ and $(1, 2, 1)$.

Compute the distance between the lines L_1 and L_2 .