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1. **(1 point)** `Library/Rochester/setLinearAlgebra12Diagonalization/ur_la_12_3.pg`

Let

$$M = \begin{bmatrix} 5 & 1 \\ -4 & 9 \end{bmatrix}.$$

Find formulas for the entries of M^n, where n is a positive integer.

$$M^n = \begin{bmatrix} \underline{\qquad\qquad\qquad} & \underline{\quad} \\ \underline{\qquad\qquad} & \underline{\quad} \end{bmatrix}$$

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2. **(1 point)** `Library/NAU/setLinearAlgebra/JordanForm.pg`

Let

$$A = \begin{bmatrix} -5 & -2 & -2 & 8 \\ 16 & 7 & 12 & -8 \\ 0 & 0 & -1 & -4 \\ -4 & -1 & -1 & 7 \end{bmatrix}.$$

Find a matrix P such that $D = P^{-1}AP$ is the Jordan canonical form of A. The Jordan form is upper triangular. The blocks are ordered increasingly by eigenvalue and then by block size.

$$P = \begin{bmatrix} \underline{\quad} & \underline{\quad} & \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} & \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} & \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} & \underline{\quad} & \underline{\quad} \end{bmatrix}, D = \begin{bmatrix} \underline{\quad} & \underline{\quad} & \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} & \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} & \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} & \underline{\quad} & \underline{\quad} \end{bmatrix}$$

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3. **(1 point)** `Library/TCNJ/TCNJ_Eigenvalues/problem1.pg`

A is an $n \times n$ matrix.

Check the true statements below:

- A. A matrix A is not invertible if and only if 0 is an eigenvalue of A.
- B. To find the eigenvalues of A, reduce A to echelon form.
- C. If $Ax = \lambda x$ for some vector x, then λ is an eigenvalue of A.
- D. Finding an eigenvector of A might be difficult, but checking whether a given vector is in fact an eigenvector is easy.
- E. A number c is an eigenvalue of A if and only if the equation $(A - cI)x = 0$ has a nontrivial solution x.

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4. **(1 point)** `Library/Rochester/setLinearAlgebra11Eigenvalues/ur_la_11_13.pg`

Suppose a 3×3 matrix A has only two distinct eigenvalues. Suppose that $\mathrm{tr}(A) = -4$ and $\det(A) = -32$. Find the eigenvalues of A with their algebraic multiplicities.

The smaller eigenvalue = ___ has multiplicity ___, and the larger eigenvalue = ___ has multiplicity ___.

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5. **(1 point)** `Library/Rochester/setLinearAlgebra11Eigenvalues/ur_la_11_9.pg`

Supppose A is an invertible $n \times n$ matrix and $\vec{v}$ is an eigenvector of A with associated eigenvalue -7. Convince yourself that $\vec{v}$ is an eigenvector of the following matrices, and find the associated eigenvalues.

(1) The matrix A^5 has an eigenvalue ____.

(2) The matrix A^{-1} has an eigenvalue ____.

(3) The matrix $A + 3I_n$ has an eigenvalue ____.

(4) The matrix $3A$ has an eigenvalue ____.

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6. **(1 point)** `Library/NAU/setLinearAlgebra/JordanBlockSizes.pg`

Let λ be an eigenvalue of the linear operator L and define $L_\lambda := L - \lambda I$. The following table lists the nullities of the powers of L_λ.

k	1	2	3
4	5	6	7
8			
nullity(L_λ^k)	6	11	16
20	24	27	30
31			

Find the sizes of the Jordan blocks corresponding to λ of the Jordan form of the matrix of L as a list of integers.

Sizes: ________

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7. **(1 point)** `Library/NAU/setLinearAlgebra/JordanForm2.pg`

Let

$$A = \begin{bmatrix} -13 & -44 & -16 & -24 \\ 0 & 1 & 0 & 0 \\ -22 & -70 & -25 & -39 \\ 24 & 76 & 28 & 43 \end{bmatrix}.$$

Find a matrix P such that $D = P^{-1}AP$ is the Jordan canonical form of A. The Jordan form is upper triangular. The blocks are ordered increasingly by eigenvalue and then by block size.

$$P = \begin{bmatrix} \underline{\quad} & \underline{\quad} & \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} & \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} & \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} & \underline{\quad} & \underline{\quad} \end{bmatrix}, D = \begin{bmatrix} \underline{\quad} & \underline{\quad} & \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} & \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} & \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} & \underline{\quad} & \underline{\quad} \end{bmatrix}$$