

Comment on Question (1)(a) from the exam.

The question was to explain the sentence “In general, we cannot push forward vector fields”. It is not difficult to give a satisfactory answer (which begins with the notion of push forward of a tangent vector), and in fact many people got full marks for their answer.

But I would like to also note a fancier interpretation of this question: does there exist a covariant functor from the category of manifolds and smooth maps to the category of vector spaces and linear maps whose action on objects is that it sends a manifold to the space of vector fields on that manifold? I expect the answer to be ‘no’. But, without additional assumptions (such as compatibility with inclusions of open subsets), I don’t know how to prove this.