Department of Mathematics, University of Toronto

Term Test 1 – November 15, 2006

MAT 137Y, Calculus!
Time Alloted: 1 hour 50 minutes

1. Evaluate the following limits. Do not use L'Hôpital's Rule to evaluate the limit.

(7%) **(i)** $\lim_{x\to 0} \frac{(x-1)^2-1}{x^2+6x}$.

(7%) **(ii)** $\lim_{t\to 0} \frac{\sin^2(5t)}{3t^2}$.

(7%) **(iii)** $\lim_{x\to 4^+} \frac{(4-x)|3x-14|}{|4-x|}$.

(7%) **(iii)** $\lim_{x\to 0} \frac{3-\sqrt{9-x^2}}{x^2}$.

2.

(7%) **(i)** Solve the inequality $\frac{x^2-3x}{x^4-1} \leq 0$. Express your answer as a union of intervals.

(ii) Suppose $\sin x = \frac{3}{4}$ and $\frac{\pi}{2} \leq x \leq \pi$. Find the exact value of each of the following expressions.

(6%) **(a)** $\tan x$.

(4%) **(b)** $\cos 2x$.

3.

(5%) **(a)** Give the precise ε, δ definition of the following statement: $\lim_{x\to a} f(x) = L$.

(12%) **(b)** Prove that $\lim_{x\to 3} \frac{x^2+1}{1-x} = -5$ directly using the precise definition of limit.

4. Consider the sequence of numbers

$$x_1 = \sqrt{1}, \quad x_2 = \sqrt{1+\sqrt{1}}, \quad x_3 = \sqrt{1+\sqrt{1+\sqrt{1}}}, \quad x_4 = \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1}}}}, \ldots,$$

so x_n containes n nested radicals and exactly n ones.

(3%) **(a)** Express x_n in terms of x_{n-1}.

(9%) **(b)** Prove for all positive integers $n \geq 2$ that x_n is irrational.

5. For each of the following statements determine whether the statement is true or false (by putting a checkmark next to the word "True" or "False") and then justifying your answer with either a proof showing that it is true, or an example showing that it is false. Failing to check either True or False for each question will result in no credit (and no part marks).

(5%) **(i)** If $g(x)$ is odd and $h(x) = (f \circ g)(x)$, then $h(x)$ is also odd. True ☐ or False ☐

(5%) **(ii)** If $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ do not exist, then $\lim_{x \to a} \Big(f(x) + g(x) \Big)$ also does not exist. True ☐ or False ☐

(5%) **(iii)** If $f(x) = x^3 - x^2 + x$, then there exists c such that $f(c) = 15$. True ☐ or False ☐

(5%) **(iv)** Suppose S is a set of real numbers and $\operatorname{lub} S$ exists. Then S has a largest element. True ☐ or False ☐

(6%) **6.** Suppose f is continuous at a and $f(a) > 1$. Prove that there exists $\delta > 0$ such that $f(x) > 1$ for all $x \in (a - \delta, a + \delta)$.