Department of Mathematics, University of Toronto MAT 137Y, 2009-10 Winter Session	Problem Set #4 Deadline: Thursday October 29, 6:10 p.m. Assignment Posted/Revised: October 21, 2009, 17:31

As a reminder, the following rules will be strictly enforced.

1. The front page must include your name, student number, your tutorial code (e.g. M3A) and the name of your teaching assistant. *Failure to put your name and/or your student number will result in a zero in your assignment. Failure to put your tutorial code and the name of your tutor will result in a 20% reduction of your assignment mark.* A cover page is not required as long as the necessary information is on the top of the first page.

2. Assignments must be submitted on letter-sized (8.5 × 11 inch) paper. *Using ripped notebook paper is unacceptable and will result in a zero in your assignment mark.* Assignments that are more than one page in length must be stapled in the top left corner. *Failure to staple such assignments will result in a 20% reduction of your assignment mark.* Do not use clear plastic binders.

3. You may hand your assignment to your instructor before the beginning of lecture, or deposit it into the MAT 137Y Assignment Box located at SS 1071 before the deadline. *Assignments handed in after 6:10 p.m. on Thursday will not be accepted for any reason, even if it is one minute late!*

4. As a courtesy to other students who are using the Math Aid Centre at SS 1071 to obtain help in other math courses, please do not occupy SS 1071 to complete your assignment on the due date.

Policy about Plagiarism. It is very helpful to have other students with whom to study, and we encourage you to work together. However, **it is extremely important that problem set solutions be written up independently, otherwise this constitutes plagiarism!** The teaching assistants will enforce this rule very strictly, and will apply severe penalties to any one in violation. In particular, the Department of Mathematics reminds all students that plagarism, cheating, and all forms of academic misconduct will not be tolerated. Students in violation of the *Code of Student Conduct* will be dealt with severely by the Department of Mathematics and the Faculty of Arts & Science.

Suggested Problems. Do not hand in.

1. SHE 2.2: 55.
2. SHE 2.3: 21, 23, 27, 31, 35, 37, 39, 45, 49, 55.
3. SHE 2.4: 5, 9, 13, 19, 27, 31, 43, 49, 51.
4. SHE 2.5: 3, 5, 11, 13, 15, 23, 43, 45, 47.
5. SHE 2.6: 1, 3, 7, 19, 21, 25, 31, 35, 39.

Required Problems. Hand in solutions to the problems below.

1. In the last problem set, we proved the existence of limits using the ε,δ definition, but how do we prove that a limit *does not* exist using the formal definition? To see this, consider an argument by contradiction: assume $\lim_{x\to a} f(x) = L$. Then for any $\varepsilon > 0$, there would exist a $\delta > 0$ such that $0 < |x-a| < \delta$ implies $|f(x)-L| < \varepsilon$. But this definition cannot possibly hold, so to show that, we find an $\varepsilon > 0$ for which regardless of any $\delta > 0$, it follows that if $0 < |x-a| < \delta$ does not imply that $|f(x)-L| < \varepsilon$. A contradiction usually occurs as a result of Theorem 2.3.1: if the limit exists, then the limit must be unique.

   In sum, to show a limit does not exist, one must show

   $$\text{There exists } \varepsilon > 0 \text{ such that for any } \delta > 0,\ \ 0 < |x-a| < \delta \text{ does not imply } |f(x)-L| < \varepsilon.$$

   Prove the following limits do not exist by the formal definition.

   $$\text{(i)}\ \lim_{x\to 0} \sin\frac{1}{x}. \qquad \text{(ii)}\ \lim_{x\to 0}\frac{1}{x^2}.$$

2. SHE 2.4: 24, 50, 54.

3. SHE 2.5: 12, 46.

4. SHE 2.6: 26.

5. Suppose $f(x)$ is a continuous function on $[0,1]$, and $0 \leq f(x) \leq 1$ for all $x \in [0,1]$.

   (a) Show that $f(x) = 1 - x$ for some number x.

   (b) Prove the more general statement: Suppose g is continuous on $[0,1]$ and $g(0) = 1$, $g(1) = 0$, then $f(x) = g(x)$ for some number x.

6. Evaluate the following limits, or show they do not exist.

   (i) $\lim_{t \to 0} \dfrac{1 - \sqrt{1+t}}{t\sqrt{1+t}}$.

   (ii) $\lim_{x \to 2} \dfrac{|x-2|}{x-2}$.

   (iii) $\lim_{x \to -1} \dfrac{x^{1/3}+1}{x^{1/5}+1}$.

   (iv) $\lim_{x \to 0} \dfrac{1 - \cos x}{2x^2}$.

   (v) $\lim_{x \to 0} \dfrac{\sin(\sin(\sin x))}{\sin x}$.

   (vi) $\lim_{x \to 0} \dfrac{\sec \frac{x}{2} - 1}{x \sin x}$.

7. A semicircle sits on top of an isoceles triangle to form a region shaped like an ice cream cone. Let θ be the angle at the base of the ice cream cone. if $S(\theta)$ is the area of the semicircle and $T(\theta)$ is the area of the triangle, find $\lim_{\theta \to 0} \dfrac{S(\theta)}{T(\theta)}$.