Department of Mathematics, University of Toronto MAT 137Y, 2009-2010 Winter Session	**Problem Set #3** **Deadline: Thursday October 15, 6:10 p.m.** Assignment Posted/Revised: October 4, 2009, 04:57

Please be reminded of the following rules:

Required Information. The front page must include your name, student number, your tutorial code (which will be assigned to you when tutorial rooms are announced), and the name of your teaching assistant. *Failure to put your name and/or your student number will result in a zero in your assignment. Failure to put the name of your TA or your tutorial code will result in a 20% reduction of your assignment mark.* A cover page is not required as long as the necessary information is on the top of the first page.

Paper Size and Requirements. Assignments must be submitted on letter-sized (8.5×11 inch) paper. *Using ripped notebook paper is unacceptable and will result in a zero in your assignment mark.* Assignments that are more than one page in length must be stapled in the top left corner. *Failure to staple such assignments will result in a 20% reduction of your assignment mark.* Do not use clear plastic binders.

Submitting your assignment. You must hand your assignment to your instructor before the beginning of lecture, or deposit the assignment into the MAT 137Y Assignment Box located inside SS 1071. *The penalty for late assignments is zero for the assignment, regardless of the excuse. Assignments handed in after 6:10 p.m. on Thursday will not be accepted for any reason, even if it is one minute late!*

Policy on Plagiarism on Assignments. It is very helpful to have other students with whom to study, and we encourage you to work together. However, **it is extremely important that problem set solutions be written up independently, otherwise this constitutes plagiarism! Don't copy other people's work, and don't let others copy your work!** The teaching assistants will enforce this rule very strictly, and will apply severe penalties to any one in violation. In particular, the Department of Mathematics reminds all students that plagiarism, cheating, and all forms of academic misconduct will not be tolerated. Students in violation of the *Code of Student Conduct* will be dealt with severely by the Department of Mathematics and the Faculty of Arts & Science.

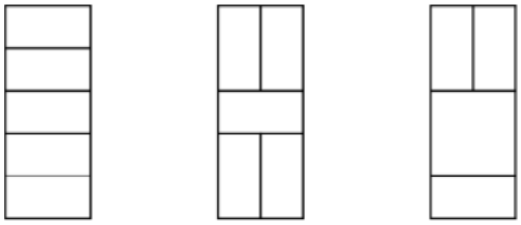
Supplementary Problems. "SHE" refers to the textbook by Salas, Hille, and Etgen (10th Edition)

1. SHE 1.2: 71, 73, 77.
2. SHE 1.8: 3, 5, 9, 11, 13.
3. SHE 2.1: 3, 5, 9, 25, 31, 33, 43, 47.
4. SHE 2.2: 5, 11, 13, 17, 23, 25, 41, 45, 49, 53, 55.

Required Problems. Hand in solutions to all the problems below.

1. (a) Let k be any integer. Prove that if k^3 is even, then k is even.
   (b) Prove that $\sqrt[3]{2}$ is irrational. Hint: Mimic the proof that $\sqrt{2}$ is irrational and apply the result from part (a).
   (c) Suppose two numbers a and b are irrational. Must the sum $a+b$ be irrational? Justify your answer with an appropriate proof or counterexample.
2. SHE 1.8: 4, 14.
3. Consider a game called NanoTetris where a *winning layout* is a complete tiling of a $2 \times n$ board using only the three shapes below.

   For example, here are some possible winning layouts on a 2×5 board.



(a) Let T_n denote the number of different winning layouts on a $2 \times n$ board. Determine the values of T_1, T_2, and T_3.

(b) Express T_n in terms of T_{n-1} and T_{n-2}. Hint: How can you construct a winning configuration for a $2 \times n$ board if you already have a winning configuration of a $2 \times (n-1)$ board? A $2 \times (n-2)$ board?).

(c) Using strong induction, prove that the number of winning layouts on a $2 \times n$ NanoTetris board (for $n \geq 1$) is given by the formula

$$T_n = \frac{2^{n+1} + (-1)^n}{3}.$$

4. Solve the following limits algebraically.

(i) $\lim_{x \to 3} \dfrac{x^2 - 6x + 9}{x^2 - x - 6}$.

(ii) $\lim_{x \to 9} \dfrac{x^2 - 81}{\sqrt{x} - 3}$.

The formal definition of limit is probably the hardest concept in calculus. We start with some exercises (5–8) that will help you understand the definition, and then conclude with limits which you must prove using the formal definition. Remember to read the handout "Proofs on Limits" which is available on the course website.

5. How close to 4 do we have to take x so that $3x+2$ is within a distance of (a) 0.1 and (b) 0.01 from 14?

6. Sketch the graph of $y = \frac{1}{x}$ for $x > 0$. Use the graph to find a number δ such that

$$\left| \frac{1}{x} - \frac{1}{2} \right| < 0.3 \text{ whenever } |x-2| < \delta.$$

7. Find an upper bound M for $f(x) = \left| \dfrac{x-2}{x+\frac{1}{2}} \right|$ if $|x+1| < \frac{1}{4}$.

8. Suppose $f(x) = x^2 + x + 1$, $a = 1$, and $L = 3$. Find a value $\delta > 0$ such that $0 < |x-a| < \delta$ implies $|f(x) - L| < \frac{1}{100}$.

9. Prove the following statements directly using the formal ε, δ definition.

(i) $\lim_{x \to 3} 5x - 4 = 11$.

(ii) $\lim_{x \to 1} \dfrac{x+3}{x^2+x+4} = \dfrac{2}{3}$.

(iii) $\lim_{x \to 1} \dfrac{x+3}{2x-1} = 4$.

(iv) $\lim_{x \to 8} \sqrt{x+1} = 3$.