Math 246S: Homework 5

Due at the beginning of tutorial
Tuesday, Feb 14, 2012 at 8:10 PM sharp!

(1) Prove that $\sqrt[3]{7}$ is irrational.

(2) Is $\sqrt{\sqrt{2}+\sqrt[3]{7}}$ irrational? Why or why not?

(3) Suppose x and y are irrational numbers. Which of the following numbers **MUST** be irrational? Justify your answer.
   (a) $x+y$
   (b) xy
   (c) $\sqrt{x+y}$
   (d) x^y

(4) Show that the equation $3x^3+200x^2-500x-2=0$ has no rational solutions. Hint: Suppose a rational solution $\frac{m}{n}$ exists (as always it is OK to assume m and n are relatively prime (why?)). What does the equation tell you about m and n?

(5) (a) Find a polynomial with integer coefficients which has $\sqrt{3}+\sqrt{5}$ as a root.
   (b) Find a polynomial with integer coefficients which has $\sqrt{3}+\sqrt{5}+\sqrt{7}$ as a root.

(6) Let α be an integer.
   (a) Find all the integer solutions of $13x+23y=0$.
   (b) Find all the integer solutions of $13x+23y=\alpha$.
   (c) Find all integer solutions of $13x+23y+z=1$.

(7) Find all integer solutions of $2x^2+3y=1$. Hint: Start by letting $z=x^2$.