Math 246S: Homework 4 Due at the beginning of tutorial Tuesday, Feb 7, 2012 at 8:10 PM sharp!

- (1) Find integers x and y so that 1352x + 495y = g.c.d(1352, 495). Can x and y be both chosen to be negative? Why or why not?
- (2) You have two glass bottles, one holds exactly 7 liters and the other 19 liters. How do you measure 1 liter of water using these two bottles?
- (3) Give a careful proof by induction that the Euclidean algorithm always allows to express g.c.d(a,b) as ax + by for some integers x, y. Hint: Use induction on the number of steps in the Euclidean algorithm.
- (4) (a) Prove that two consecutive natural numbers (for example 2013 and 2012) are relatively prime.
 - (b) Compute the multiplicative inverse of 2013 mod 2012 using the Euclidean algorithm.
- (5) Find an integer x satisfying the following equation: $2x \equiv 19 \pmod{77}$.
- (6) Find a multiple of 77 that ends with 999.
- (7) Find all integer solutions of 13x + 23y = 1.
- (8) You are to receive a message using the RSA system. You choose p = 7, q = 19 and e = 5. You therefore tell the world that N is 133 and e is 5. I send you an encoded message; the encoded version you receive is 100. What is the original (before coding) message?
- (9) Consider the RSA system. Suppose N and e are given. For a message M < N, the encoded message is denoted by R. As M ranges over all the integers between 0 and N 1 what are all the numbers R that you get? Hint: R is an integer between 0 and N 1.