

Math 246S: Homework 2
Due at the beginning of tutorial
Tuesday, Jan 24, 2012 at 8:10 PM sharp!

- (1) Prove that $\frac{n^5-5n^3+4n}{120}$ is an integer for all $n \in \mathbb{N}$.
Hint: factor the numerator.
- (2) Let p be a prime and a an integer not divisible by p .
Prove: $ax \equiv ay \pmod{p^2}$ implies $x \equiv y \pmod{p^2}$.
- (3) Determine whether or not $17^{2492} + 25^{376} + 5^{782}$ is divisible by 3.
Prove that your answer is correct.
- (4) (a) Find a solution to the congruence $2x \equiv 9 \pmod{5}$.
(b) Find a solution to the congruence $2x \equiv 9 \pmod{10}$.
- (5) Fix a natural number n . Prove that $(1+n)^{3m} - 1$ is divisible by n for every $m \in \mathbb{N}$.
- (6) Give a proof by induction that $a \equiv b \pmod{m}$ implies $a^n \equiv b^n \pmod{m}$ for any $n \geq 1$.
- (7) Fix a natural number $m > 1$. Prove by induction that for any integer $n \geq 0$ there exists a natural number r such $0 \leq r < m$ and $n \equiv r \pmod{m}$.
- (8) Show that there is no natural number k such that $2^k \equiv 1 \pmod{6}$.
Find all possible values of $2^k \pmod{6}$.