## Math 246S: Homework 1 <br> Due at the beginning of tutorial Tuesday, Jan 17, 2012 at 8:10 PM sharp!

(1) Prove by induction that any natural number is either even or odd.
(2) Let $r \neq 1$ be a real number. Prove by induction that for all natural numbers $n$,

$$
1+r+r^{2}+\cdots+r^{n}=\frac{1-r^{n+1}}{1-r}
$$

(3) Prove that the principle of complete mathematical induction implies the principle of mathematical induction.
(4) Prove by induction that for all natural numbers $n$ the following formula holds:

$$
\frac{1}{1 \cdot 2}+\frac{1}{2 \cdot 3}+\cdots+\frac{1}{n(n+1)}=\frac{n}{n+1}
$$

(5) Let $x_{1}>2$. Define $x_{n}$ by the formula $x_{n+1}=\frac{3 x_{n}+2}{x_{n}+2}$. Prove by induction that $x_{n}>2$ for all natural numbers $n$.
(6) Find and prove a formula for the sum $1^{3}+2^{3}+\cdots+n^{3}$.
(7) Prove the Fundamental Theorem of Arithmetic (Theorem 4.1 of "Beautiful Mathematics") using Corollary 4.3

