Math 246S: Homework 1 Due at the beginning of tutorial Tuesday, Jan 17, 2012 at 8:10 PM sharp!

- (1) Prove by induction that any natural number is either even or odd.
- (2) Let $r \neq 1$ be a real number. Prove by induction that for all natural numbers n,

$$1 + r + r^{2} + \dots + r^{n} = \frac{1 - r^{n+1}}{1 - r}.$$

- (3) Prove that the principle of *complete* mathematical induction implies the principle of mathematical induction.
- (4) Prove by induction that for all natural numbers n the following formula holds:

$$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \dots + \frac{1}{n(n+1)} = \frac{n}{n+1}.$$

- (5) Let $x_1 > 2$. Define x_n by the formula $x_{n+1} = \frac{3x_n+2}{x_n+2}$. Prove by induction that $x_n > 2$ for all natural numbers n.
- (6) Find and prove a formula for the sum $1^3 + 2^3 + \cdots + n^3$.
- (7) Prove the Fundamental Theorem of Arithmetic (Theorem 4.1 of "Beautiful Mathematics") using Corollary 4.3