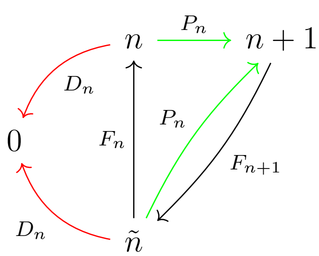
Analysis of the star force enhancement system for superior equips

We provide a way to compute the expected number of enhancements, $h_{i,j}$, to go from i stars to j stars $(0 \leq i < j < 10)$ for superior equipment. We also provide a way to compute the expected number of destroyed equips, $d_{i,j}$, while going from i stars to j stars.

Let P_i denote the probability of successfully enhancing at state i. Let D_i denote the probability of destroying an item while attempting to enhance at state i. Let $F_i := 1 - D_i - P_i$ denote the probability of demotion or stability while at state i.

Our model for the enhancements is described by the following two diagrams

$$F_0 \circlearrowleft 0 \xrightarrow{P_0} 1 \xrightarrow{P_1} \cdots \xrightarrow{P_8} 9 \xrightarrow{P_9} 10 \circlearrowleft$$



$(n = 1, \ldots, 8).$ $(*)$

The role of the first diagram should be apparent, so we will only explain the role of the latter diagram. The state $\tilde{n}$ serves to act as state n while also remembering that it arrived at state n from having backtracked once from the last attempted enhancement. If one succeeds from state $\tilde{n}$, then one is promoted to state $n+1$, just as it would have been from state n. If one fails at state $\tilde{n}$, then one is sent to state n. This is feature not quite like chance time, because there is a missing step in reaching state n from a failure at state $\tilde{n}$. This is remedied in the recurrence relations below.

For the remainder of this document, we fix the following ordering of states:

$$(n_k) := (n_1, \ldots, n_{18}) := (0, 1, \tilde{1}, \ldots, 8, \tilde{8}, 9).$$

Let M be the Markov chain corresponding to the above diagram according to this order.

Expected number of enhancements from i stars to 10 stars

We first calculate $h_{i,10}$ for $0 \leq i < 10$. The recurrence relations for these values are as follows

$$\begin{cases} h_{0,10} = \frac{1}{2}(1+h_{0,10}) + \frac{1}{2}(1+h_{1,10}) \\ h_{1,10} = \frac{1}{2}(1+h_{0,10}) + \frac{1}{2}(1+h_{2,10}) \\ h_{i,10} = F_i(1+h_{\widetilde{i-1},10}) + S_i(1+h_{i+1,10}) + D_i(1+h_{0,10}) \quad (2 \leq i \leq 9) \\ h_{\tilde{i},10} = F_i(2+h_{i,10}) + S_i(1+h_{i+1,10}) + D_i(1+h_{0,10}) \quad (1 \leq i \leq 8) \\ h_{10,10} = 0 \end{cases} . \quad (1)$$

Let $h := (h_{n_k,10})^t$ and let

$$u := ((1,\ldots,1) + (0,0,P_{\tilde{1}},0,P_{\tilde{2}},\ldots,0,P_{\tilde{8}},0))^t.$$

Then the recurrence relations (1) are given by

$$(I-M)h = u \iff h = (I-M)^{-1}u \quad (1^*)$$

and we find that

$$\begin{aligned} h_{0,10} &= 285.846 & h_{5,10} &= 249.089 \\ h_{1,10} &= 283.846 & h_{6,10} &= 230.325 \\ h_{2,10} &= 279.946 & h_{7,10} &= 204.421 \\ h_{3,10} &= 273.345 & h_{8,10} &= 167.176 \\ h_{4,10} &= 263.158 & h_{9,10} &= 109.986 \end{aligned}$$

These values were rounded to the nearest third decimal place.

Expected number of destroyed items from i stars to 10 stars

We now compute $d_{i,10}$ for $0 \leq i < 10$. The recurrence relations for the expected number of destroyed items are as follows

$$\begin{cases} d_{0,10} = \frac{1}{2}d_0 + \frac{1}{2}d_1 \\ d_{1,10} = \frac{1}{2}d_0 + \frac{1}{2}d_2 \\ d_{i,10} = F_i d_{\widetilde{i-1}} + S_i d_{i+1} + D_i(1 + d_{0,10}) \quad (2 \leq i \leq 9) \\ d_{\tilde{i},10} = F_i d_i + S_i d_{i+1} + D_i(1 + d_{0,10}) \qquad (1 \leq i \leq 8) \\ d_{10,10} = 0 \end{cases} . \tag{2}$$

Define $d := (d_{n_k,10})^t$, let

$$v := (D_{n_k})^t = (0, ..., 0, D_5, D_{\tilde{5}}, \ldots, D_8, D_{\tilde{8}}, D_9)^t,$$

and lastly, let M_0 denote the Markov chain M with its destruction entries removed (that is, the entirety of the first column of M is removed with the exception of the (1,1) and (2,1) entry). Then the recurrence relations (2) are given by

$$(I - M_0)d = v \iff d = (I - M_0)^{-1}v. \tag{2*}$$

We find now that

$$\begin{aligned} d_{0,10} &= 2.09 & d_{5,10} &= 2.09 \\ d_{1,10} &= 2.09 & d_{6,10} &= 2.045 \\ d_{2,10} &= 2.09 & d_{7,10} &= 1.903 \\ d_{3,10} &= 2.09 & d_{8,10} &= 1.615 \\ d_{4,10} &= 2.09 & d_{9,10} &= 1.093 \end{aligned}$$

These values were rounded to the nearest third decimal place.

Going from $h_{i,10}$ to $h_{i,j}$ and from $d_{i,10}$ to $d_{i,j}$

If you look at the definitions of $h_{i,j}$ and $d_{i,j}$, it's plain that

$$h_{i,j} = h_{i,10} - h_{j,10} \quad \text{and} \quad d_{i,j} = d_{i,10} - d_{j,10}$$

for $0 \leq i < j < 10$.

An example

We may calculate the number of destructions $d_{8,9}$ expected while enhancing from 8 stars to 9 stars. As mentioned above, this is given by

$$d_{8,9} = d_{8,10} - d_{9,10} = .522.$$

Similarly, we find that the expected number of enhancements from 8 stars to 9 is

$$h_{8,9} = 57.19.$$

The value $d_{8,9}$ is not at odds with the fact that the probability of destruction going from 8 stars to 9 stars is .06. The value $h_{8,9}$ indicates that the expected number of attempts at enhancement from 8 to 9 is around 57. Consider a fixed path of length 57 going from 8 stars to 9 stars for sake of concreteness. This path taken going from 8 stars to 9 stars will rack up possibilities of destruction at many places (possibly all at going from 8 stars to 9 stars, but more likely other places as well).