The expected cost of enhancing regular equipment

We will provide a formula for the expected cost of enhancing regular (that is, equipment which are not superior) equipment from i to j stars which is only a function of the base cost of enhancing from 0 to 1 star (and thus applicable to equipments of varying levels). This takes into account both chance time and the possibility of destruction which would then result in starting back from 0. Lastly, we provide the expected number of destroyed items while enhancing from 0 to 15 stars.

Since enhancement costs for regular equipment vary at different stages, one will not be able to calculate the expected cost from knowing the expected number of steps from i stars to j stars alone. A slightly modified notion, which we will call a *weighted step* will allow for calculation of the expected cost from i stars to j stars which we will use $\mathrm{EC}(i, j)$ to mean. Define the *weighted step function*

$$\omega(k) := \begin{cases} k+1 & \text{if } 0 \leq k < 10 \\ 2(k-10)^2 + 18(k-10) + 81 & \text{if } 10 \leq k < 15. \end{cases}$$

We claim that

$$\mathrm{EC}(k, k+1) = \mathrm{EC}(0,1) * \omega(k). \tag{$*$}$$

This can be checked against the data available here. Note that $(*)$ is to be understood as a close approximation rather than an exact formula. Another important note is that $\omega(k)$ is slightly off for the values 12,13,14. Better values for $\omega(k)$ for $k = 12, 13$, and 14, respectively would be 126, 154, 185 rather than 125, 153, 185, which is what our formula gives. We chose to sacrifice some precision to simplify matters slightly.

The results were obtained by using three different Markov chains: one for 0 stars $\mapsto$ 5 stars, one for 5 stars $\mapsto$ 10 stars, and another for 10 stars $\mapsto$ 15 stars. We will not provide a detailed description of how we chose the models, because it is not only similar to the calculation conducted here, but also simpler.

The results

We present the number of weighted steps $\Omega(i,j)$ it takes to get from i stars to j stars.

$$\Omega(0,5) = 17.76, \quad \Omega(5,10) = 103.91, \quad \Omega(10,15) = 19758.57.$$

As we mentioned at the beginning, the reason for introducing weighted steps is the following:

$$\text{EC}(i,j) = \Omega(i,j) * \text{EC}(0,1).$$

One might object at this point (or have been rightfully have been objecting this whole time): We have been slightly abusing notation this whole because EC is a function of the level of the equipments at hand.

We give an example application to level 160 equips. We will be using the cost of the 150 equips in the first link since complete data for 160 equips is not available (and the cost of the 150 equips given in that link are higher than the cost of the cost for 160 equips in GMS).

$$\begin{aligned} \text{EC}(10,15) &= \Omega(10,15) * \text{EC}(0,1) \\ &= 19758.57 * .136 \\ &= 2687.17 \text{ million mesos} \\ &\simeq 2.687 \text{ billion mesos.} \end{aligned}$$

The expected number of destructions $\text{ED}(i,j)$ going from i to j stars was found to be

$$\text{ED}(12,15) = .385, \quad \text{ED}(13,15) = .352, \quad \text{ED}(14,15) = .241.$$

Note that $\text{ED}(i,15) = \text{ED}(12,15)$ for $0 \leq i < 12$ because destructions do not occur from i stars to $i+1$ stars. So in particular, $\text{ED}(0,15) = .385$.

Thanks to Beastied for pointing out that my initial results were implausible.