MAT1000HF Fall 2017

Midterm Practice Problems 3

Problem 1

Let $(X, \mathcal{M}, \mu)$ be σ-finite and $(f_n)_{n\in\mathbb{N}}$ be a sequence of measurable functions $f_n : X \to \mathbb{C}$; show that there exists a sequence of positive real numbers $(c_n)_{n\in\mathbb{N}}$ so that

$$\sum_{n=1}^{\infty} c_n f_n$$

converges almost everywhere.

Problem 2

Prove that $L^1(\mu) \cap L^\infty(\mu)$ is dense in $L^p(\mu)$ for $1 \leq p < \infty$. Is the same true for $p = \infty$?

Problem 3

Let E be a Borel set and $f : E \to \mathbb{R}$ be in $L^1(m)$; show that if $\int_{B\cap E} f dm = 0$ for any ball B centered at points of E, then $f = 0$ μ-a.e. on E.

Problem 4

Recall the definition of the Cantor function $F : [0, 1] \to [0, 1]$, which is the only increasing continuous function of $[0, 1]$ onto itself that is constant equal to $(2k - 1)/2^n$ on the k-th gap (counted from left to right) removed at the n-th generation in the construction of the standard (middle-third) Cantor set $C \subset [0, 1]$. Let μ_F be the associated Lebesgue–Stiltjes measure on $[0, 1]$. Compute $\mu_F(C)$.