MAT415: HOMEWORK SET #5

DUE: MONDAY APRIL 13, 2015

Let L be a finite extension of $\mathbb{Q}_p$ with $\mathcal{O}_K$ the integral closure of $\mathbb{Z}_p$, and set π to be a unformizer of $\mathcal{O}_L$. For a non-zero element α, define $v_L(\alpha)$ to be the integer $m \in \mathbb{Z}$ such that $\alpha \in \pi^m \mathcal{O}_L$, and $|\alpha|_L := |k_L|^{-v_K(\alpha)}$. Additionally, define $v_L(0)$ to be ∞, thought of as positive infinity.

(1) Recall that an infinite sum converges if its partial sums form a cauchy sequence. Prove for a sequence $(a_i)_{i>0}$ of L that $\sum_{i=1}^{\infty} a_i$ converges iff the sequence itself converges to 0. In other words, the sum converges iff $\lim_{i\to\infty} |a_i|_L = 0$.

(2) Prove the power series $\exp(x) = \sum_{i=0}^{\infty} \frac{x^i}{i!}$ converges for $|x|_L < |p|_L^{\frac{1}{p-1}}$.

(3) Suppose $p > 2$. Prove that $\mathbb{Z}_p^{\times}$ is isomorphic, as a topological group, to $\frac{\mathbb{Z}}{(p-1)\mathbb{Z}} \times \mathbb{Z}_p$. (That is, there is a continuous group isomorphism between the two groups with a continuous inverse).

(4) Determine all the quadratic extensions of $\mathbb{Q}_2$.