Welcome to MAT137!

(Section L5201, Thursdays 6-9pm in MS3153)

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Welcome to MAT137!

- Course website: http://uoft.me/MAT137
- Make sure you have read and understood the course outline.
  (To find it, go to: Course website → Resources.)
- Make sure you check your mail.utoronto.ca email regularly for announcements.
  (Google "utmail mobile" for info on how to set up phone/tablet access, or just go to this link.)
- Join Piazza, our online help forum.
  (For links, go to: Course website → Resources.)

Welcome to MAT137!

- My page for just our section.
  (To find it, go to: Course website → Resources.)

  Please get into the habit of both the course website and the page above for announcements.

  In particular, our section's page will tell you which videos to watch before each lecture. Don't leave it to the last minute!

- Precalculus review: `http://uoft.me/precalc`
  (Strong precalc skills are more important than previous knowledge of calculus.)

  Going through this is "Problem Set 0".

How did students do two years ago?

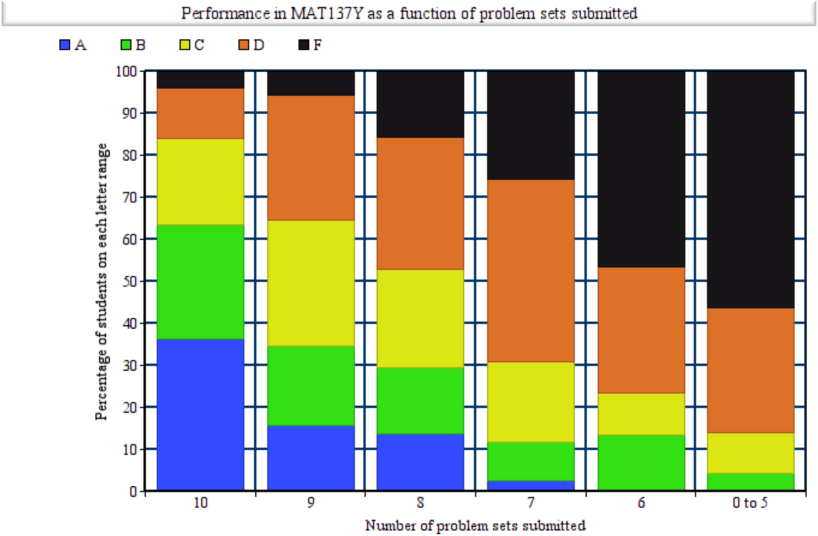
MAT137 results from 2015 - 2016:

(among students who wrote the final exam)

# of submitted problem sets	A	A or B	F
10	35%	58%	4%
9	19%	41%	9%
8	5%	22%	22%
5 to 7	1%	9%	45%
Fewer than 5	2%	2%	79%

How did students do last year?

MAT137 results from 2016 - 2017:



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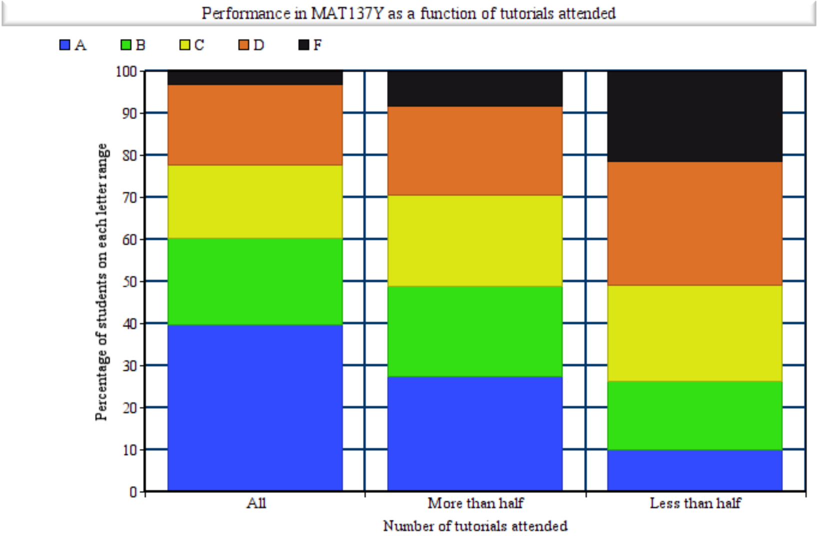
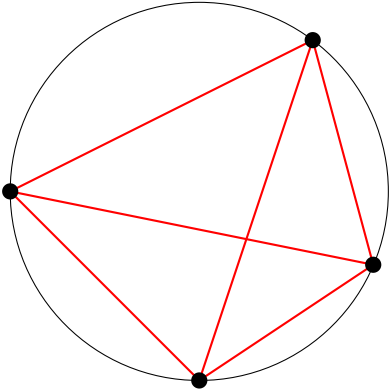


Proofs are important

- Pick 4 points at random on a circle (not necessarily evenly spaced).
- Join every pair of points.
- Into how many regions is the circle divided?



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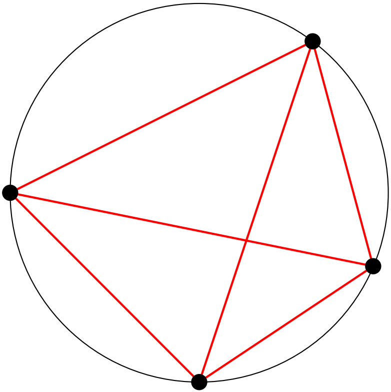

# of points	# of regions
1	1
2	2
3	4
4	8
5	16
6	??
n	??

Actual formula: $\frac{1}{24}(n^4 - 6n^3 + 23n^2 - 18n + 24)$.

(Proving this is hard.)

Another famous example

Consider the function

$$\pi(x) = \# \text{ of prime numbers less than or equal to } x.$$

For example:

$$\begin{array}{cc} \pi(2) = 1 & \pi(10) = 4 \\ \pi(3) = 2 & \pi(11) = 5 \\ \pi(4) = 2 & \pi(100) = 25 \end{array}$$

This function is *extremely important* to number theorists, but it is not very well understood. A much simpler function, called $\operatorname{li}(x)$ was proved to approximate $\pi(x)$ quite well in 1896.

Another famous example

For all integers n that anyone has ever checked (even to this day), we have found that

$$\pi(n) - \mathrm{li}(n) < 0.$$

In other words, $\mathrm{li}(n)$ always seems to *overestimate* $\pi(n)$.

There is literally no numerical evidence that $\mathrm{li}(n)$ ever underestimates $\pi(n)$, even for a single value of n.

However, J. E. Littlewood proved in 1914 that $\pi(n) - \mathrm{li}(n)$ switches sign *infinitely many times* as n increases!

The earliest estimate (made in 1955) for the first place the sign changes was on the order of $10^{10^{10^{964}}}$, which is an outrageously large number. We've since improved this bound to around 1.4×10^{316}.

Warm-up exercise (get it?)

Consider the following statement:

"No two students in this class are not on fire."

Question: Which of the following statements are equivalent to it?

1. "All student in this class, except at most one, are on fire."
2. "Two students in this class are on fire."
3. "For any pair of students in this class, one of them is on fire."
4. "At least two students in this class are not on fire."
5. "If I choose two students in this class and one of them is not on fire, then the other one is on fire."

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Which of them (if any) is equivalent to its negation?

Sets

Describe the following sets in the simplest terms you can.

1. $[2,4] \cup (3,10)$
2. $[2,4] \cap (3,10)$
3. $(\pi, 3)$
4. $[7,7]$
5. $(7,7)$
6. $A = \{\, x \in \mathbb{R} \,:\, x^2 < 7 \,\}$
7. $B = \{\, x \in \mathbb{Z} \,:\, x^2 < 7 \,\}$
8. $C = \{\, x \in \mathbb{N} \,:\, x^2 < 7 \,\}$

Some more notation for sets

Given two sets A and B, we define:

- $A \setminus B = \{ x \in A \, : \, x \notin B \}$.
  We usually read this as "A without B" or similar. It's the set consisting of all things in A that are not in B.

- $A \triangle B = (A \setminus B) \cup (B \setminus A)$.
  We usually read this as "the symmetric difference between A and B". It's the set of all things in A or B but not both.

To check your understanding of notation, convince yourself that

$$A \triangle B = (A \cup B) \setminus (A \cap B).$$

Using the notation we just defined

Problem 1. Define the following two sets:

- $A = \{\text{male students in this class}\}$
- $B = \{\text{students sitting in the first two rows}\}$

What are the sets $A \setminus B$, $B \setminus A$, and $A \triangle B$?

Some negations

Write down the negations of the following statements as simply as you can:

1. Every student in this room has a cellphone.
2. There is a province in Canada with fewer than 1000 inhabitants.
3. Ivan likes coffee and tea.
4. Every building at UofT contains a classroom with no windows.

Quantifiers and the empty set

Are the following statements true or false?

- There is a purple giraffe in this room.
- All giraffes in this room are purple.

Domination

(This is a problem from a previous year's first problem set.)

If A and B are both sets of real numbers, we say B *dominates* A if the following is true:

For every $a \in A$, there exists $b \in B$ such that $a < b$.

If you prefer mathematical notation:

$$\forall a \in A, \exists b \in B, \text{ such that } a < b.$$

Problem. Find two non-empty sets of real numbers A and B such that the following three things are true:

1. $A \cap B = \emptyset$.
2. A dominates B.
3. B dominates A.