MAT137 - Week 3

- Problem Set 1 is due tomorrow, at 3pm. The deadline is *firm*.
  - You should have received an email with a personal link to submit it. You *must* use that link, and should not share it.
  - If you haven't seen that email, check your spam folder.
  - If you still can't find it, contact Alfonso. All of this is on the course website.
  - **Do not leave the submission to the last minute.**

- Today's lecture is about limits.
  - The intuitive idea of a limit.
  - Some review material about absolute values.
  - The formal definition of a limit.

Informal definition of a limit

This definition is not precise, but it will guide us to write the precise definition later.

Definition

Let f be a function, and let a and L be real numbers. We say

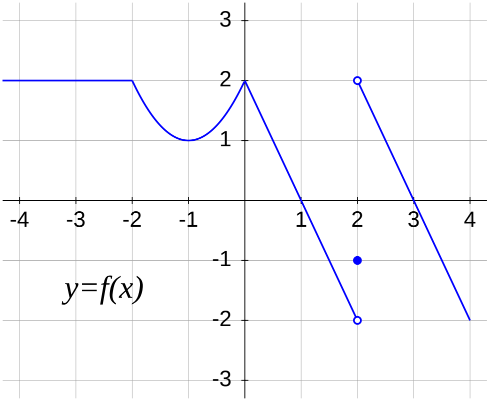
the limit of f as x approaches a is L

if $f(x)$ can be made arbitrarily close to L simply by making x sufficiently close to (but not equal to) a.

If this definition is satisfied, we will write:

$$\lim_{x \to a} f(x) = L$$

Limits from a graph



Find the value of

1. $\lim_{x \to 2} f(x)$
2. $\lim_{x \to 0} f(f(x))$

Towards the precise definition of a limit

Let's take our informal definition earlier, and translate it into mathematical notation.

Definition

$\lim_{x \to a} f(x) = L$ means

$$\forall \epsilon > 0 \, \exists \delta > 0 \text{ such that } 0 < |x - a| < \delta \implies |f(x) - L| < \epsilon.$$

Towards the precise definition of a limit

Translation into normal English:

$\forall \epsilon > 0$	"No matter how close you require me to get, ..."
$\exists \delta > 0$ such that	"...there is a distance δ such that..."
$0 < \lvert x - a \rvert < \delta \Longrightarrow$	"...if x is within δ of (but not equal to) a, then..."
$\lvert f(x) - L \rvert < \epsilon$	"...$f(x)$ is closer to L than you required."

The correct definition of a limit

Our last definition still isn't good enough, because it doesn't say what a or L or f are. Here's the final, correct definition.

Definition

Let f be a function defined on an open interval containing a real number a, except possibly at a. Let L be a real number. Then $\lim_{x \to a} f(x) = L$ means

$$\forall \epsilon > 0 \, \exists \delta > 0 \text{ such that } 0 < |x - a| < \delta \implies |f(x) - L| < \epsilon.$$