MAT137 - Week 2

- Course website: `http://uoft.me/MAT137`
- Remember: Tutorials start next week.
  - Attend the tutorial in which you're enrolled.
  - If you have to change tutorials, instructions are on the website.
  - Check Portal, not ROSI/ACORN for your tutorial.
- Join Piazza, our online help forum. Seriously, it's great.
- Next week we will go back to a more traditional, lecture style.
- Reminder that Problem Set 1 is available, and due 30 September.
  - Next week you'll get an email invitation to submit it online.

Some negation exercises

Write down the negations of the following statements statements as simply as you can:

1. Every student in this room has a cellphone.
2. There is a province in Canada with fewer than 1000 inhabitants.
3. Ivan likes coffee and tea.
4. Every building at UofT contains a classroom with no windows.
5. If Ivan likes tea, then he likes coffee too.
6. If a UofT student likes tea, then they like coffee too.

Negating conditional statements

Here's a great way of thinking about negating conditional statements. Suppose I made you the following promise:

If you get an A (or better) in MAT137, I will give you cake.

Under what circumstances would I have lied to you? Under what circumstances would I have kept my word? For example, if...

- ...you get a C, and I don't give you cake?
- ...I just give everyone cake?
- ...you get an A+, and I don't give you cake?

...more negation.

Negate the following statement without using any negative words ("no", "not", "none", etc.):

> *"Every page in this book contains at least one word whose first and last letters both come alphabetically before M."*

Evens and odds

Write down formal definitions for what it means for an integer to be even or odd.

Evens and odds

Write down formal definitions for what it means for an integer to be even or odd.

Which of the following is a correct definition for "odd"?

1. x is odd if $x = 2n + 1$.
2. x is odd if $\forall n \in \mathbb{Z}$, $x = 2n + 1$.
3. x is odd if $\exists n \in \mathbb{Z}$ such that $x = 2n + 1$.

Evens and odds

Write down formal definitions for what it means for an integer to be even or odd.

Which of the following is a correct definition for "odd"?

1. x is odd if $x = 2n + 1$.
2. x is odd if $\forall n \in \mathbb{Z}$, $x = 2n + 1$.
3. x is odd if $\exists n \in \mathbb{Z}$ such that $x = 2n + 1$.

Having established the definition of oddness, evenness is easy and similar:

x is even if $\exists n \in \mathbb{Z}$, $x = 2n$.

Evens and odds (continued)

Consider the following theorem:

Theorem

The sum of two odd integers is even.

What are some things wrong with the following "proof"?

Proof.

$x = 2a + 1$
$y = 2b + 1$
$x + y = 2n$
$(2a + 1) + (2b + 1) = 2n$
$2(a + b + 1) = 2n$
$a + b + 1 = n.$

□

Evens and odds (continued)

What about the following proof:

Proof.

For all n:

EVEN + EVEN = EVEN

EVEN + ODD = ODD

ODD + ODD = EVEN □

Write a proof for this statement that is less awful.

Definitions - Injectivity

A function f defined on a domain D is called *injective on* D (or *one-to-one on* D) if different inputs to the function always yield different outputs.

For example, $f(x) = x$ is injective, while $f(x) = x^2$ is not. You can see this from their graphs easily.

Write down a formal definition for this property.

Definitions - Injectivity (continued)

Here are some candidates.

For all of these, suppose f is a function defined on a nonempty domain D.

1. $f(x_1) \neq f(x_2)$.
2. $\forall x_1, x_2 \in D,\ x_1 \neq x_2,\ f(x_1) \neq f(x_2)$.
3. $\exists x_1, x_2 \in D$ such that $x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$.
4. $\forall x_1, x_2 \in D,\ f(x_1) \neq f(x_2) \implies x_1 \neq x_2$.
5. $\forall x_1, x_2 \in D,\ f(x_1) = f(x_2) \implies x_1 = x_2$.
6. $\forall x_1, x_2 \in D,\ x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$.

Recall how induction works...

To prove a statement S_n is true for all $n \geq 1$, you can do the following two things:

1. **Base case:** Prove that S_1 (or some other starting point) is true.
2. **Induction hypothesis:** Prove that $\forall n \geq 1$,

$$S_n \text{ is true} \Longrightarrow S_{n+1} \text{ is true.}$$

Induction

Suppose we have some statements S_n for all $n \geq 1$.

In each of the following cases, which S_n's will we know are true?

1. **Case 1:** Suppose we have shown that:
   - S_7 is true.
   - $\forall n \geq 1$, S_n is true $\implies S_{n+1}$ is true.
2. **Case 2:** Suppose we have shown that:
   - S_1 is true.
   - $\forall n \geq 7$, S_n is true $\implies S_{n+1}$ is true.
3. **Case 3:** Suppose we have shown that:
   - S_1 is true.
   - $\forall n \geq 1$, S_{n+1} is true $\implies S_n$ is true.
4. **Case 4:** Suppose we have shown that:
   - S_1 is true.
   - $\forall n \geq 1$, S_n is true $\implies S_{n+3}$ is true.