Welcome to MAT137!

(Section L5201, Thursdays 6-9pm in MS3153)

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Welcome to MAT137!

- Course website: `http://uoft.me/MAT137`
- Make sure you have read the syllabus/course outline.
- Make sure you check your mail.utoronto.ca email regularly for announcements.
- Join Piazza, our online help forum.
- Precalculus review: `http://uoft.me/precalc`
- Homework before next week's class: Watch all of the remaining videos.

How did students do last year? (or, Do your homework!)

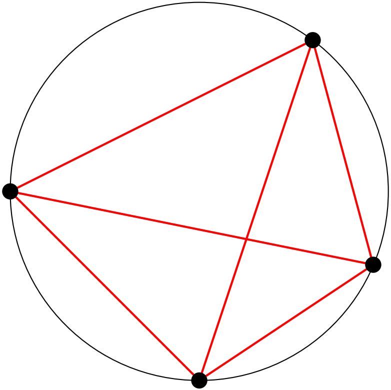
MAT137 results from 2015 - 2016:

(among students who wrote the final exam)

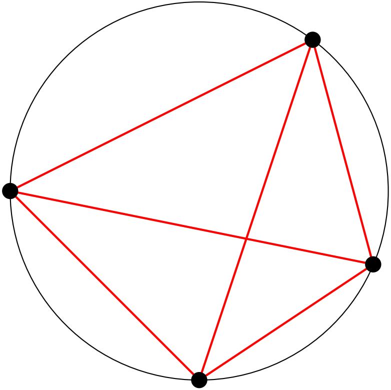
# of submitted problem sets	A	A or B	F
10	35%	58%	4%
9	19%	41%	9%
8	5%	22%	22%
5 to 7	1%	9%	45%
Fewer than 5	2%	2%	79%

Proofs are important

- Pick 4 points at random on a circle (not necessarily evenly spaced).
- Join every pair of points.
- Into how many regions is the circle divided?

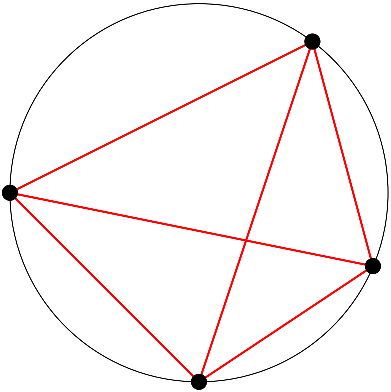


Proofs are important



# of points	# of regions
1	
2	
3	
4	8
5	

Proofs are important



# of points	# of regions
1	1
2	2
3	4
4	8
5	16
6	30 or 31
n	??

Actual formula: $\frac{1}{24}(n^4 - 6n^3 + 23n^2 - 18n + 24)$.

(Proving this is hard.)

Another famous example

Consider the function

$$\pi(x) = \# \text{ of prime numbers less than or equal to } x.$$

For example:

$$\begin{array}{rr} \pi(2) = 1 & \pi(10) = 4 \\ \pi(3) = 2 & \pi(11) = 5 \\ \pi(4) = 2 & \pi(100) = 25 \end{array}$$

This function is *extremely important* to number theorists, but it is not very well understood. A much simpler function, called $\text{li}(x)$ was proved to approximate $\pi(x)$ quite well in 1896.

Another famous example

For all integers n that anyone has ever checked (even to this day), we have found that

$$\pi(n) - \mathrm{li}(n) < 0.$$

In other words, $\mathrm{li}(n)$ always seems to *overestimate* $\pi(n)$.

There is literally no numerical evidence that $\mathrm{li}(n)$ ever underestimates $\pi(n)$, even for a single value of n.

However, Littlewood proved in 1914 that $\pi(n) - \mathrm{li}(n)$ switches sign *infinitely many times* as n increases!

The earliest estimate (made in 1955) for the first place the sign changes was on the order of $10^{10^{10^{964}}}$. We've since improved this to about 1.4×10^{316}.

Sets

Describe the following sets in the simplest terms you can.

1. $[2, 4] \cup (3, 10)$
2. $[2, 4] \cap (3, 10)$
3. $(\pi, 3)$
4. $[7, 7]$
5. $(7, 7)$
6. $A = \{ x \in \mathbb{R} : x^2 < 7 \}$
7. $B = \{ x \in \mathbb{Z} : x^2 < 7 \}$
8. $C = \{ x \in \mathbb{N} : x^2 < 7 \}$

Some more notation for sets

Given two sets A and B, we define:

- $A \setminus B = \{ x \in A : x \notin B \}$.
  We usually read this as "A without B" or similar. It's the set consisting of all things in A that are not in B.

- $A \triangle B = (A \setminus B) \cup (B \setminus A)$.
  We usually read this as "the symmetric difference between A and B". It's the set of all things in A or B but not both.

To check your understanding of notation, convince yourself that

$$A \triangle B = (A \cup B) \setminus (A \cap B).$$

Some set problems

Problem 1. Define the following two sets:

- $A = \{\text{male students in this class}\}$
- $B = \{\text{students sitting in the first two rows}\}$

What are the sets $A \setminus B$, $B \setminus A$, and $A \triangle B$?

Problem 2. A real number that is not rational is called *irrational*.

Let A be the set of all negative, rational numbers and positive, irrational numbers.

Write a definition of A using only mathematical notation. (There is more than one way to do this.)

Sets defined with quantifiers

Problem 1. Describe the following sets in the simplest terms you can.

1. $A = \{\, x \in \mathbb{R} \,:\, \forall y \in [5,7],\ x < y \,\}$.
2. $B = \{\, x \in \mathbb{R} \,:\, \exists y \in [5,7] \text{ such that } x < y \,\}$
3. $C = \{\, x \in [5,7] \,:\, \forall y \in [5,7],\ x < y \,\}$.
4. $D = \{\, x \in [5,7] \,:\, \exists y \in [5,7] \text{ such that } x < y \,\}$
5. $E = \{\, x \in [5,7] \,:\, \exists y \in \mathbb{R} \text{ such that } x < y \,\}$
6. $F = \{\, x \in [5,7] \,:\, y \in \mathbb{R},\ x < y \,\}$

Quantifiers and the empty set

Are the following statements true or false?

- There is a purple giraffe in this room.
- All giraffes in this room are purple.