One general strategy for such problems: try to experiment with small values of \( n \) and look for patterns.

1. Alice and Bob play the following game. They start with a pile of 9 matches. They take turns, Alice playing first. Each player may remove between 1 and 3 matches. The player who picks up the last match wins. Who has a winning strategy? And what is it?

2. Same problem as before, but the player who takes the last match loses. Who has a winning strategy now? And what is it?

3. In both of the above problems, what if the players start with \( n \) matches? That is, who has a winning strategy, depending on \( n \)?

4. Another variation of the first problem: now the players may take \( 2^d \) matches for any \( d \geq 0 \). If they start with \( n \) matches and the player who takes the last match wins, who has a winning strategy and what is it? (depending on \( n \))

5. Two players A and B play the following game. A thinks of a polynomial with non-negative integer coefficients. B must guess the polynomial. B has two shots: she can pick a number and ask A to return the polynomial value there, and then she has another such try. Can B win the game?

6. Alan and Barbara play a game in which they take turns filling entries of an initially empty \( 2008 \times 2008 \) array. Alan plays first. At each turn, a player chooses a real number and places it in a vacant entry. The game ends when all the entries are filled. Alan wins if the determinant of the resulting matrix is nonzero; Barbara wins if it is zero. Which player has a winning strategy?

7. Suppose there are \( n \) lights in a row. Each is equipped with a switch, which changes that lamp and its neighbouring lamps from on to off or vice versa. (So operating the switch at one of the ends changes two lamps, any other switch changes three lamps.) For which \( n \) is it guaranteed that all lamps can be switched off, no matter which lamps are on initially.
8. Alice and Bob play a game in which they take turns removing stones from a heap that initially has \( n \) stones. The number of stones removed at each turn must be one less than a prime number. The winner is the player who takes the last stone. Alice plays first. Prove that there are infinitely many \( n \) such that Bob has a winning strategy. (For example, if \( n = 17 \), then Alice might take 6 leaving 11; then Bob might take 1 leaving 10; then Alice can take the remaining stones to win.)