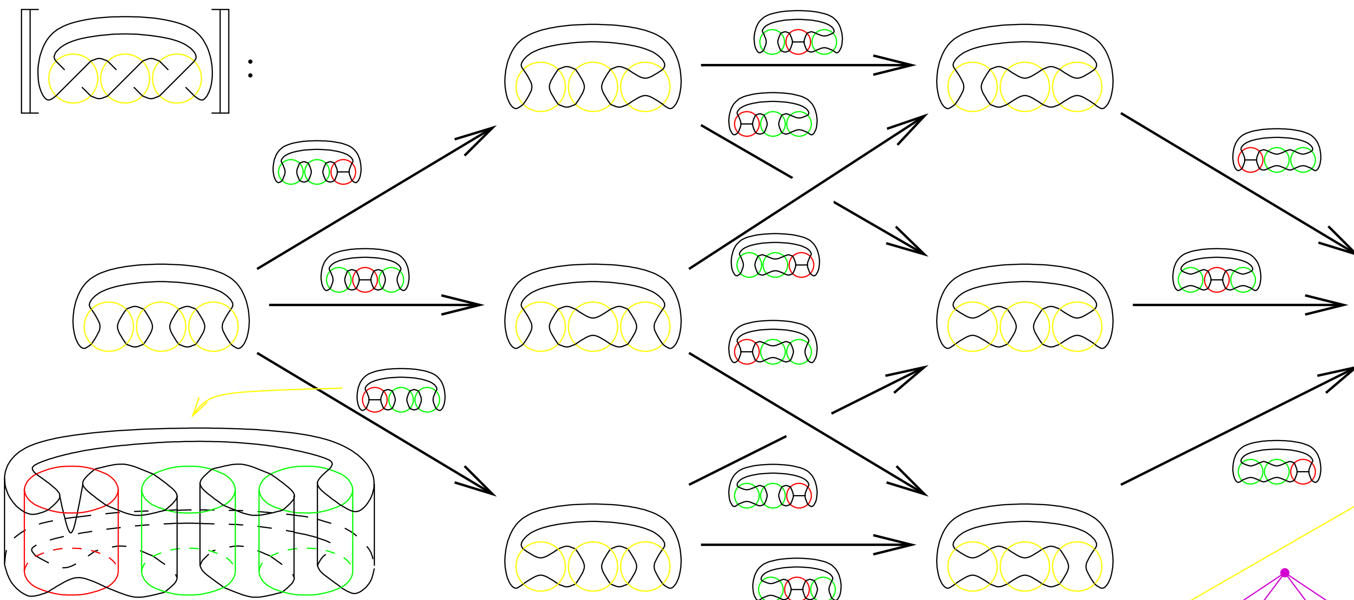
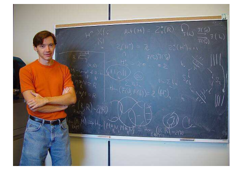
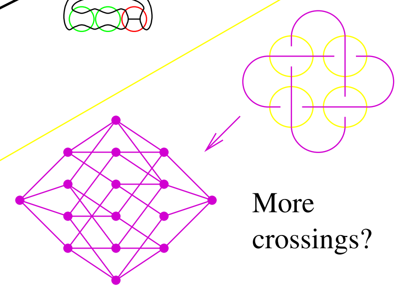
Khovanov Homology





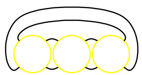
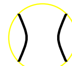
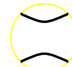
Mikhail Khovanov

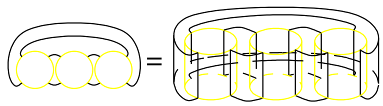
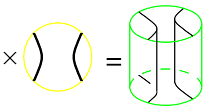
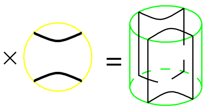


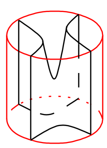
More crossings?

What is it?

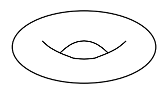
A cube for each knot/link projection;

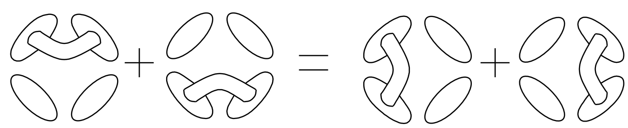
Vertices: All fillings of  with  or with .

Edges: All fillings of $I\times$  with $I\times$  or with $I\times$  and

with precisely one .

Kom: Complexes
Mat: Matrices
<...>: Formal linear combinations
Cob: Cobordisms

S:  *=0* *T:*  *=2*

4Tu: 

Where does it live?

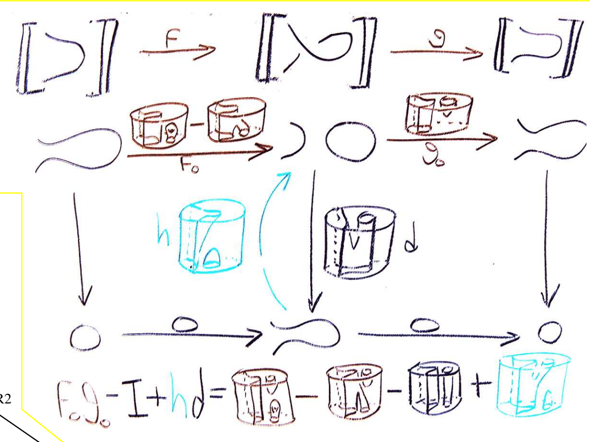
In *Kom(Mat(<Cob> / {S, T, 4Tu}) / homotopy* .

It looks like Jones!

Indeed, a TQFT takes it to a complex whose graded Euler characteristic is the Jones polynomial.

But is it invariant?

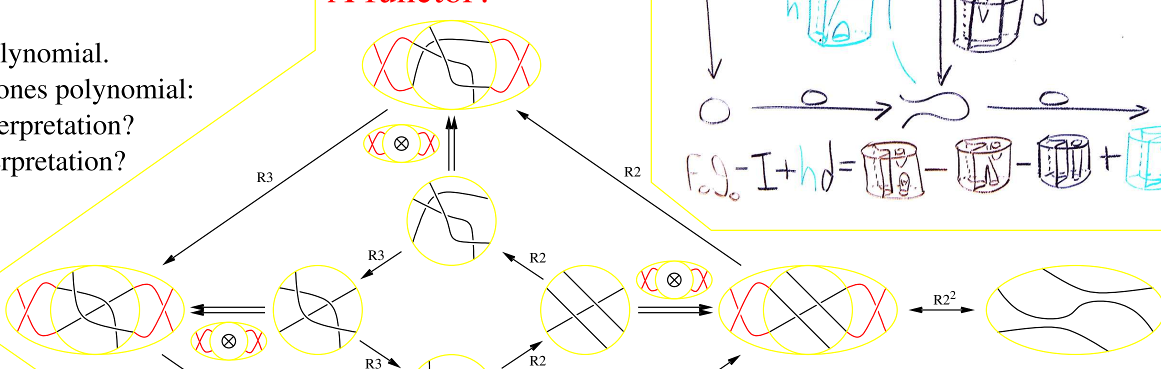
(With similar proofs for R–II and R–III)



Why is it intersting?

1. It is stronger than the Jones polynomial.
2. It is less understood than the Jones polynomial:
   a. Does it have a topological interpretation?
   b. Does it have a "physical" interpretation?
   c. Does it also work for other quantum invariants?
   d. Does it work for manifolds and for knots in manifolds?
   e. Is there a relation with finite–type invariants?
   f. Does it work for "virtual knots"?
3. Jacobsson, Khovanov: It is a functor!!! (from knots and cobordisms to complexes and morphisms)

A functor?



See http://www.math.toronto.edu/~drorbn/papers/Categorification

A Canopoly?

Dror Bar–Natan, April 2003, Toronto under the Plague