Do not turn this page until instructed.

Math 327 Introduction to Topology

Term Test

University of Toronto, October 28, 2010

Solve 4 of the 5 problems on the other side of this page.
Each problem is worth 25 points.
You have an hour and fifty minutes to write this test.

Notes.

- No outside material other than stationary is allowed.
- **Neatness counts! Language counts!** The *ideal* written solution to a problem looks like a page from a textbook; neat and clean and made of complete and grammatical sentences. Definitely phrases like "there exists" or "for every" cannot be skipped. Lectures are mostly made of spoken words, and so the blackboard part of proofs given during lectures often omits or shortens key phrases. The ideal written solution to a problem does not do that.
- Advance apology: It may take us a while to grade this exam; sorry.

Good Luck!

Solve 4 of the following 5 problems. Each problem is worth 25 points. You have an hour and fifty minutes. **Neatness counts! Language counts!**

Problem 1. An "open map" $f\colon X \to Y$ from a topological space X to a topological space Y is a function for which if U is open in X, then $f(U)$ is open in Y.

1. Let X and Y are topological spaces and let $X \times Y$ be their product. Show that the projection on the first factor, $\pi_X\colon X \times Y \to X$, is an open map.
2. Is the function $f(x) = x^2$, defined on the real line $\mathbb{R}$ and taking values in the real line $\mathbb{R}$, an open map?

Tip. In math-talk, "show" means "prove".
Tip. In math exams, yes/no answers must be accompanied with a proof.

Problem 2. Let A be a subset of a topological space X. For a point $x \in X$, prove that $x \in \bar{A}$ if and only if for every neighborhood U of x, the intersection $U \cap A$ is not empty.
Tip. "If and only if" means that there are two things to prove.

Problem 3. Let (X, d) be a metric space.

1. Show that the function $d\colon X \times X \to \mathbb{R}$ is continuous.
2. Show that the metric topology of X is the weakest topology for which the previous statement is true. In other words, show that if X' is X taken with some other topology, and if $d\colon X' \times X' \to \mathbb{R}$ is continuous, then the topology on X' is stronger than the topology on X.

Problem 4. Let $(\bar{x}_n)$ be a sequence in $\mathbb{R}^\omega$ (so $\bar{x}_n = (x_{n,1}, x_{n,2}, \ldots)$, with $x_{n,k} \in \mathbb{R}$), and let $\bar{y} = (y_1, y_2, \ldots)$ be a point of $\mathbb{R}^\omega$. Prove that $\lim_{n\to\infty} \bar{x}_n = \bar{y}$ (using the product topology on $\mathbb{R}^\omega$) if and only if for every k, $\lim_{n\to\infty} x_{n,k} = y_k$.

Problem 5. Prove in detail that $\mathbb{R}^\omega$ is not connected in the box topology.

Good Luck!