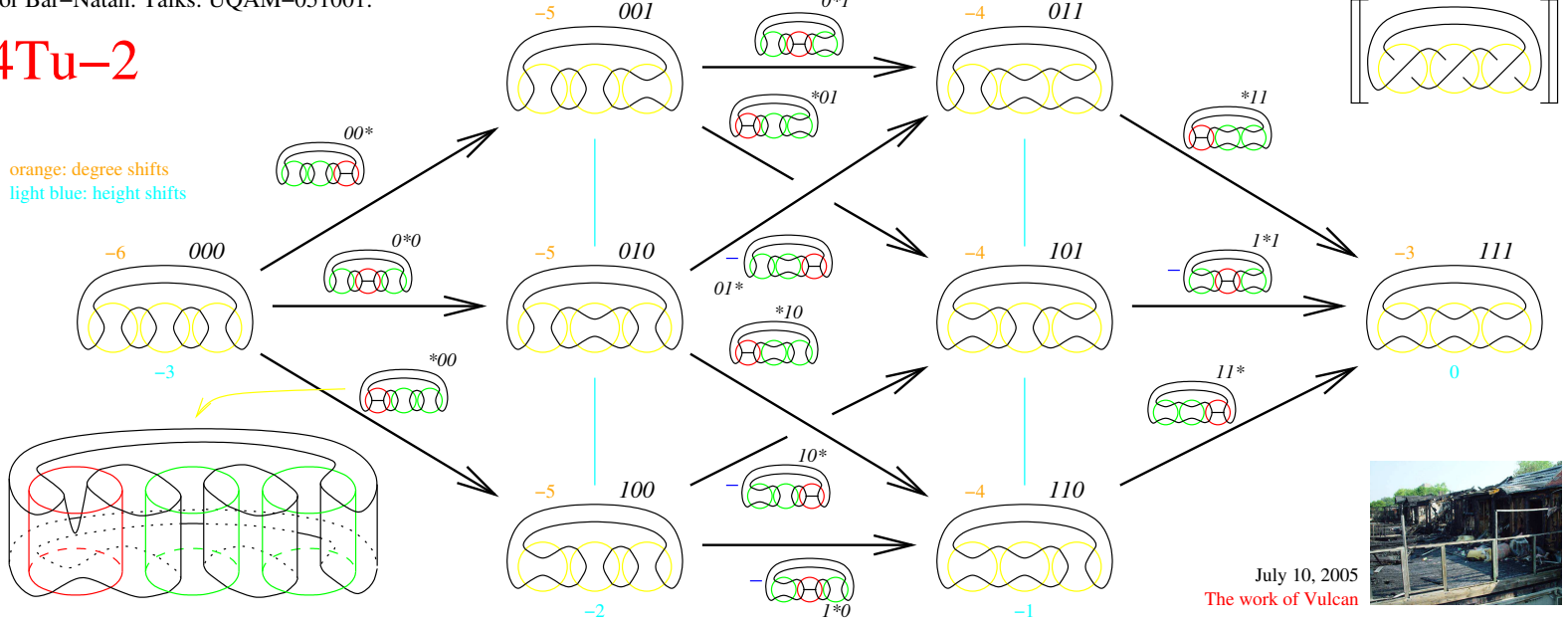
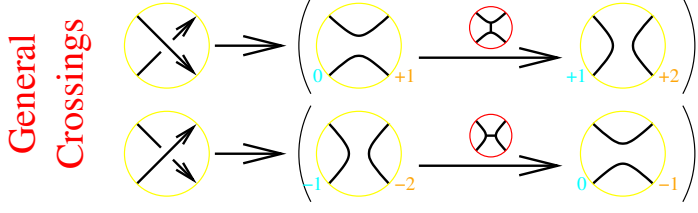


# 4Tu-2

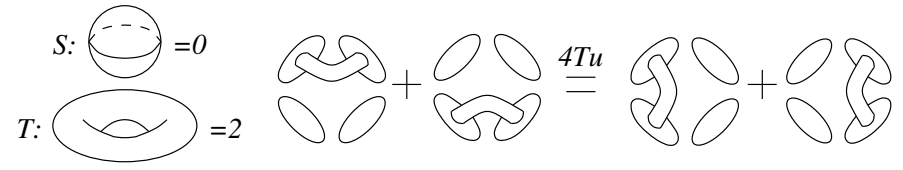
orange: degree shifts  
light blue: height shifts



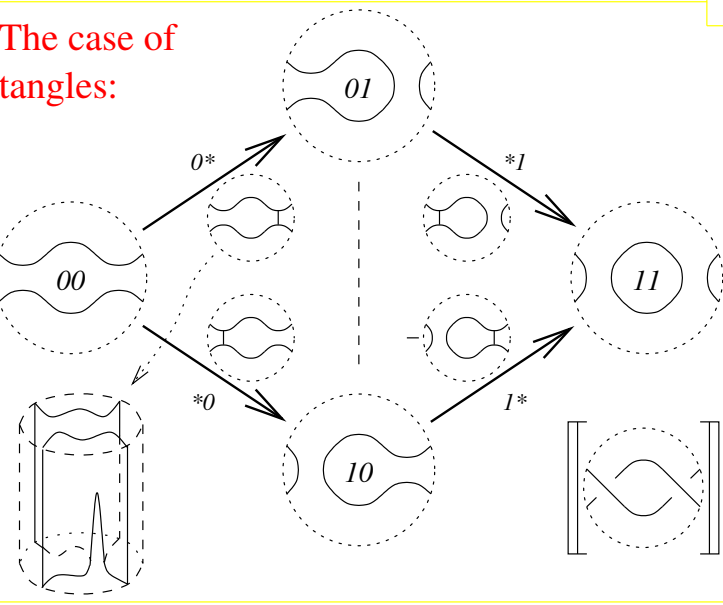
July 10, 2005  
The work of Vulcan



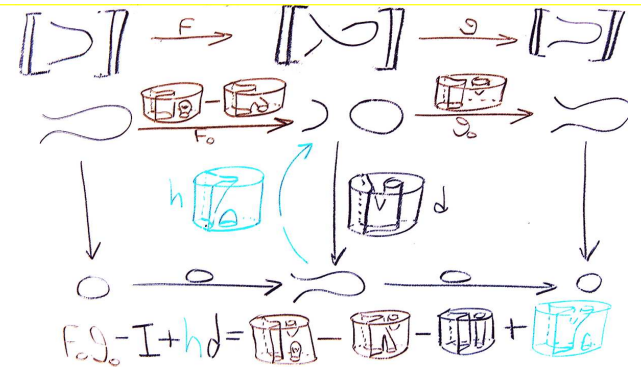
**Where does it live?** In  $Kom(Mat(\langle Cob \rangle / \{S, T, 4Tu\})) / \text{homotopy}$   
 Kom: Complexes Mat: Matrices Cob: Cobordisms  $\langle \dots \rangle$ : Formal lin. comb.



### The case of tangles:



### Invariant!



**The Reduction Lemma.** If  $\phi$  is an isomorphism then the complex

$$[C] \xrightarrow{\begin{pmatrix} \alpha \\ \beta \end{pmatrix}} \begin{bmatrix} b_1 \\ D \end{bmatrix} \xrightarrow{\begin{pmatrix} \phi & \delta \\ \gamma & \epsilon \end{pmatrix}} \begin{bmatrix} b_2 \\ E \end{bmatrix} \xrightarrow{\begin{pmatrix} \mu & \nu \end{pmatrix}} [F]$$

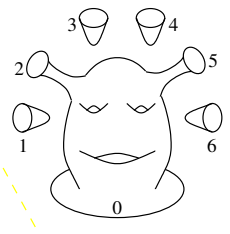
is isomorphic to the (direct sum) complex

$$[C] \xrightarrow{\begin{pmatrix} 0 \\ \beta \end{pmatrix}} \begin{bmatrix} b_1 \\ D \end{bmatrix} \xrightarrow{\begin{pmatrix} \phi & 0 \\ 0 & \epsilon - \gamma\phi^{-1}\delta \end{pmatrix}} \begin{bmatrix} b_2 \\ E \end{bmatrix} \xrightarrow{\begin{pmatrix} 0 & \nu \end{pmatrix}} [F]$$

### The work of Naot.

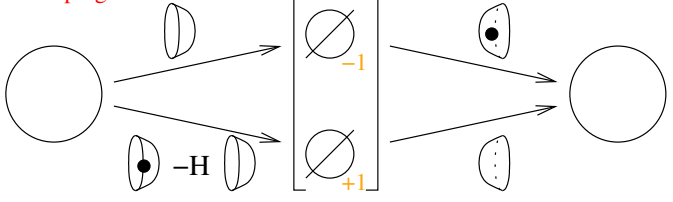
$\langle \text{surfaces} \rangle / 4Tu$  is freely generated by Shrek surfaces

A Shrek surface with 7 boundaries (one distinguished), 3 handles and 2 tubes



Let  $\bullet$  denote a tube to the distinguished component (the curtain), and let H denote a handle on the curtain. Then

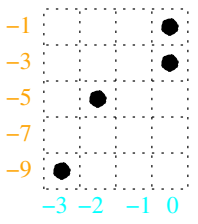
### Delooping:



... so the invariant is valued in complexes over a category with just one object and morphisms in  $Z[H]$ ; all is graded and  $\text{deg}H = -2$ .

### The work of Green.

standard data:

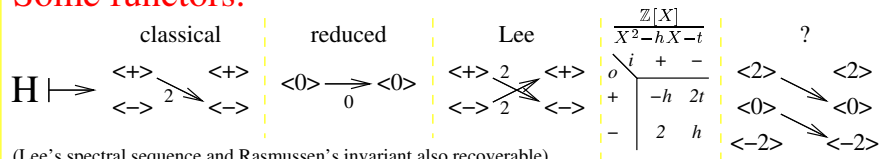


The universal invariant of the left-handed trefoil is

$$\begin{array}{c} \begin{array}{|c|} \hline \text{Trefoil} \\ \hline \end{array} \\ \xrightarrow{\quad} \begin{array}{|c|} \hline H \\ \hline \end{array} \xrightarrow{\quad} 0 \xrightarrow{\quad} \begin{array}{|c|} \hline \end{array} \\ \begin{array}{|c|} \hline -3 \\ \hline \end{array} \begin{array}{|c|} \hline -8 \\ \hline \end{array} \begin{array}{|c|} \hline -2 \\ \hline \end{array} \begin{array}{|c|} \hline -6 \\ \hline \end{array} \begin{array}{|c|} \hline -1 \\ \hline \end{array} \begin{array}{|c|} \hline 0 \\ \hline \end{array} \begin{array}{|c|} \hline -2 \\ \hline \end{array} \end{array}$$

(and the invariant of the 48 crossing  $T(8,7)$  is computable in minutes...)

### Some functors.



(Lee's spectral sequence and Rasmussen's invariant also recoverable)