

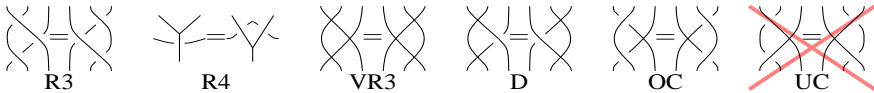
Trivalent (framed) w-tangles:

further operations: delete, unzip.

$$wTT = CA \langle \text{trivalent diagrams} \rangle / R123, R4 \text{ (for vertices)}, F, OC.$$

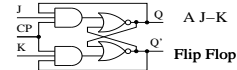
$$= PA \langle \text{trivalent diagrams} \rangle / R1234, F, VR1234, D, OC.$$

(=tangles in thick surfaces, modulo stabilization)

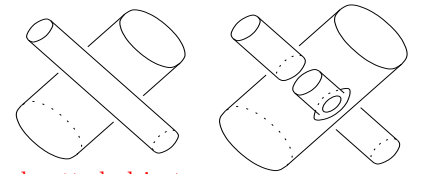
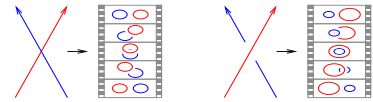


Circuit Algebras

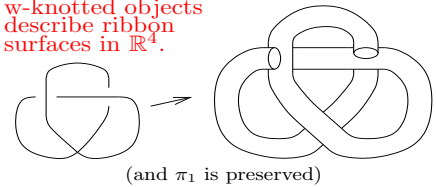
- * Have "circuits" with "ends"
- * Can be wired arbitrarily.
- * May have "relations" – de-Morgan, etc.



w-braids describe flying rings:



w-knotted objects describe ribbon surfaces in \mathbb{R}^4 .



(and π_1 is preserved)

Partial Dictionary.

$$(R, F) \leftrightarrow (\text{trivalent diagrams}, \text{trivalent diagrams}) \quad (r, t) \leftrightarrow (\text{trivalent diagrams}, \text{trivalent diagrams})$$

$$R^{12}R^{13}R^{23} = R^{23}R^{13}R^{12} \leftrightarrow \text{trivalent diagrams} = \text{trivalent diagrams}$$

$$FF^! = I \leftrightarrow \text{trivalent diagrams} \xrightarrow{\text{unzip}} \text{trivalent diagrams}$$

$$F^{-1}e(x+y)F = e(x)e(y)$$

$$F^{23}R^{123} = R^{12}R^{13}F^{23} \leftrightarrow \text{trivalent diagrams} = \text{trivalent diagrams}$$

$$R^{12,3} = R^{13}R^{23}$$

$$F^{123}R^{12,3} = R^{13}R^{23}F^{12,3} \leftrightarrow \text{trivalent diagrams} = \text{trivalent diagrams}$$

(unforbidding FI makes this automatic)

$$RF^{21}e(-t) = F \leftrightarrow \text{trivalent diagrams} = \text{trivalent diagrams}$$

For the Experienced (and sharp-eyed)

The "Chord Diagrams" — A_n^{wt} . As we did for quandles, substitute into the various moves, to get relations. Also switch to arrow diagram language: $\text{trivalent diagrams} \leftrightarrow \text{trivalent diagrams}$. Get:
 $R3 \mapsto \text{trivalent diagrams} = \text{trivalent diagrams}$ (tails commute)
 $R4 \mapsto \text{trivalent diagrams} = \text{trivalent diagrams}$ (really FI)
 $R4 \mapsto \text{trivalent diagrams} = \text{trivalent diagrams}$ (vertex invariance)

The "Jacobi Diagrams" — A_n^{cc} . **Theorem.** A_n^{wt} is A_n^{cc} is $U(\text{tder}_n)$. Here A_n^{cc} is trivalent directed trees with only 2-in 1-out vertices. In tensorland, this is "Co-commutative Lie-bialgebras".
 Rules: tails commute. Heads satisfy the only possible STs: + also IHX and vertex invariance.

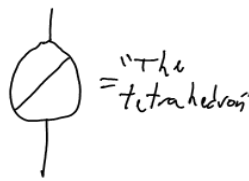
The Map $\alpha: A_n^{tree} \rightarrow A_n^{cc}$. **Theorem.** α is an injection on $A_n^{tree} \cong U(\text{sder}_n)$. Furthermore, there is a simple characterization of $\text{im } \alpha$, so we can tell "an arrowless element" when we see it.

The Main Theorem. (approximate, false as stated) F 's in Sol_0^T are in a bijective correspondance with tree-level associators for ordinary parenthesized tangles (or ordinary knotted trivalent graphs) / with homomorphic expansions for trivalent w-tangles / with solutions of the Kashiwara-Vergne problem.

Extra. Restricted to knots, we get precisely the Alexander polynomial.

Disclaimer. Orientations, rotation numbers, framings, the vertical direction and the cyclic symmetry of the vertex may still make everything uglier. I hope not.

$$\Phi = (F^{12,3})^{-1} (F^{12})^{-1} F^{2,3} F^{1,2,3} \leftrightarrow$$

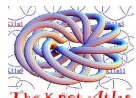


The pentagon and The hexagons Follow, with a minor twist, from The fact that we have an unzip behaved invariant of KTG's.

"God created the knots, all else in topology is the work of mortals"



Leopold Kronecker (paraphrased)



Visit! Edit!
<http://katlas.org>

This handout and further links are at <http://www.math.toronto.edu/~drorbn/Talks/MSRI-0808/>