

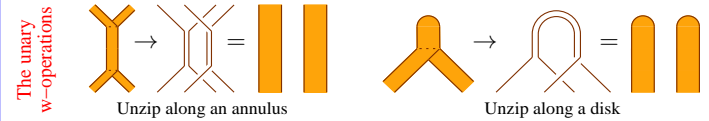
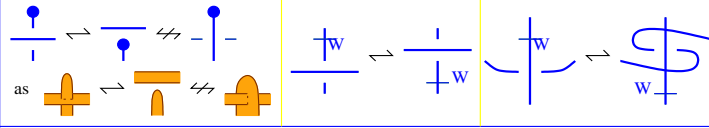
Facts and Dreams About v-Knots and Etingof-Kazhdan, 2

Example 6 - Ribbon 2-Knots.

Also, "movies of flying rings".



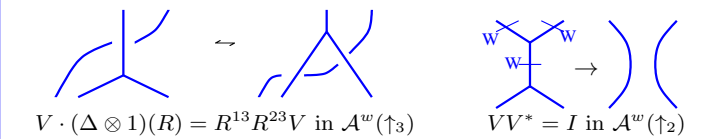
The w -relations include R234, VR1234, D, Overcrossings Commute (OC) but not UC:



Trivalent w-Tangles.

$$wTT = PA \left\langle \begin{array}{c} w- \\ \text{generators} \end{array} \middle| \begin{array}{c} w- \\ \text{relations} \end{array} \middle| \begin{array}{c} \text{unary } w- \\ \text{operations} \end{array} \right\rangle = CA \left\langle \begin{array}{c} \text{same} \\ w/o \times \end{array} \right\rangle$$

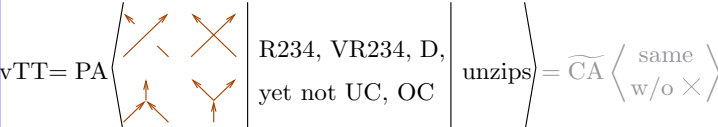
Theorem. There exists a homomorphic expansion Z for wTT . In particular, Z respects R4 and intertwines annulus and disk unzips:



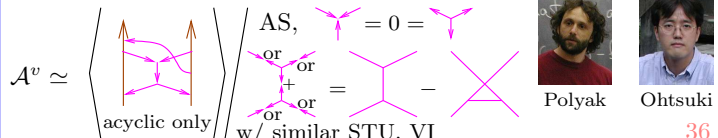
Alekseev-Torossian [AT] (equivalent to Kashiwara-Vergne [KV]). There are elements $F \in \text{TAut}_2$ and $a \in \mathfrak{t}_1$ such that

$$F(x+y) = \log e^x e^y \quad \text{and} \quad jF = a(x) + a(y) - a(\log e^x e^y).$$

Theorem. That's equivalent to a homomorphic expansion for wTT .



The Polyak-Ohtsuki Description of \mathcal{A}^v [Po].



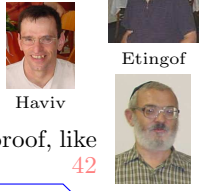
\mathcal{A}^v pairs with Lie bialgebras. Let \mathfrak{g}_+ be a Lie bialgebra with basis X_a , bracket $[\cdot, \cdot]$, cobracket δ , dual $\mathfrak{g}_- = \mathfrak{g}_+^*$, dual basis X^a for \mathfrak{g}_- , double $\mathfrak{g} = \mathfrak{g}_+ \oplus \mathfrak{g}_-$, structure constants $[X_a, X_b] = \sum b_{ab}^c X_c$ and co-structure constants $\delta(X_a) = \sum c_a^{bc} X_b \otimes X_c$. Then

$$\sum_{a,b,c,d,e,f=1}^{\dim \mathfrak{g}} b_{de}^c b_a^{ba} X_a X^d X_f \otimes X_b X^e X^c \in \mathcal{U}(\mathfrak{g})^{\otimes 2}$$

www.katlas.org The Knot Atlas

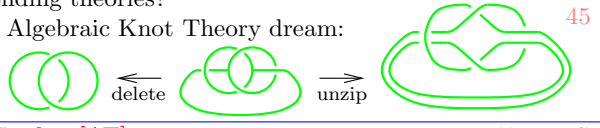
Forbidden Theorem [EK, Ha, ?]. There exists a homomorphic expansion Z for vTT .

- Why Forbidden (to me)?**
- Minor statement details may be off.
 - No fully written proof.
 - I don't understand the proof.
 - There isn't yet a knot-theoretic view of the proof, like there is in the w -case.



Why Should We Care?

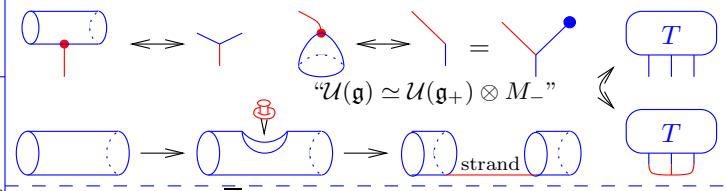
- A gateway into the forbidden territory of "quantum groups".
- Abstractly more pleasing: We study the things, and not just their representations.
- \mathcal{A}^v is sometimes easier than \mathcal{A}^u : Alexander, say, arises easily from the 2D Lie algebra⁴.
- Potentially, \mathcal{A}^v has many more "internal quotients" than there are Lie bialgebras. What are they and what are the corresponding theories?
- My old⁵ Algebraic Knot Theory dream:



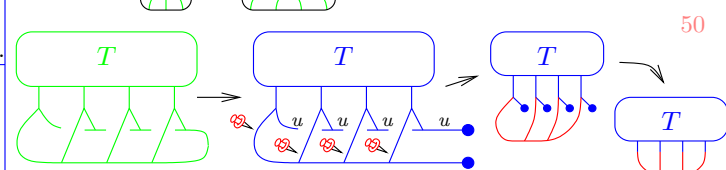
$V \rightarrow \Phi$ -loop after [AT]. "cut and cap" is well-defined(!) on \mathcal{K}^u



$\Phi \rightarrow V$ after [AET]. In \mathcal{K}^w allow tubes and tube-strand vertices, allow "punctures", yet allow no "tangles".



The generators of \mathcal{K}^w can be written in terms of the generators of \mathcal{K}^u (i.e., given Φ , can write a formula for V). With T any classical tangle, esp. $\left(\begin{array}{c} \cap \\ \cup \end{array} \right)$ or $\left(\begin{array}{c} \cup \\ \cap \end{array} \right)$, consider the "sled"



Alexander is easy! In Chicago, [BN4] Many kinds of virtuals!

Help Needed!