

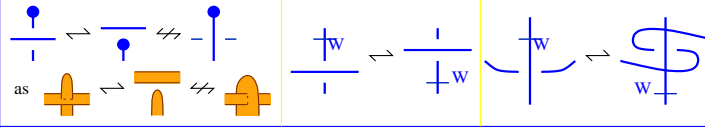
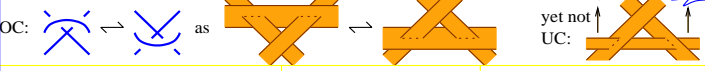
**Facts and Dreams About v-Knots and Etingof-Kazhdan, 2**

**Example 6 - Ribbon 2-Knots.**

Also, "movies of flying rings".



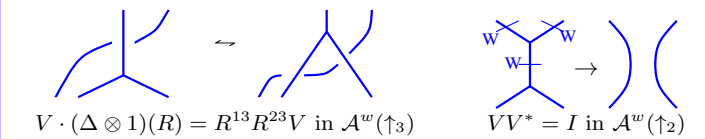
The w-relations include R234, VR1234, D, Overcrossings Commute (OC) but not UC:



**Trivalent w-Tangles.**

$$wTT = PA \left\langle \begin{array}{c} w- \\ \text{generators} \end{array} \middle| \begin{array}{c} w- \\ \text{relations} \end{array} \middle| \begin{array}{c} \text{unary } w- \\ \text{operations} \end{array} \right\rangle = CA \left\langle \begin{array}{c} \text{same} \\ w/o \times \end{array} \right\rangle$$

**Theorem.** There exists a homomorphic expansion Z for wTT. In particular, Z respects R4 and intertwines annulus and disk unzips:

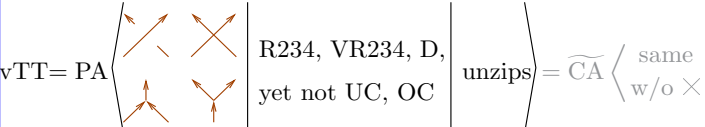


**Alekseev-Torossian [AT]** (equivalent to Kashiwara-Vergne [KV]). There are elements  $F \in \text{TAut}_2$  and  $a \in \mathfrak{t}_1$  such that

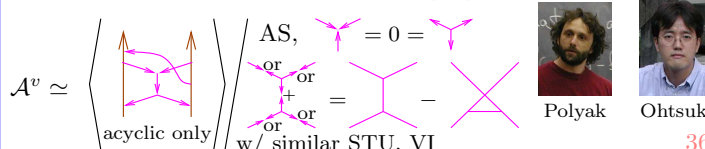
$$F(x+y) = \log e^x e^y \quad \text{and} \quad jF = a(x) + a(y) - a(\log e^x e^y).$$

**Theorem.** That's equivalent to a homomorphic expansion for wTT.

**The Main Example.**

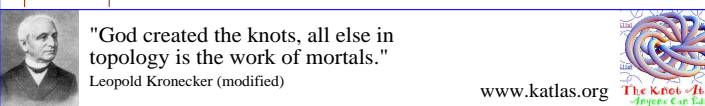


**The Polyak-Ohtsuki Description of  $\mathcal{A}^v$  [Po].**



$\mathcal{A}^v$  pairs with Lie bialgebras. Let  $\mathfrak{g}_+$  be a Lie bialgebra with basis  $X_a$ , bracket  $[\cdot, \cdot]$ , cobracket  $\delta$ , dual  $\mathfrak{g}_- = \mathfrak{g}_+^*$ , dual basis  $X^a$  for  $\mathfrak{g}_-$ , double  $\mathfrak{g} = \mathfrak{g}_+ \oplus \mathfrak{g}_-$ , structure constants  $[X_a, X_b] = \sum b_{ab}^c X_c$  and co-structure constants  $\delta(X_a) = \sum c_{ab}^c X_b \otimes X_c$ . Then

$$\sum_{a,b,c,d,e,f=1}^{\dim \mathfrak{g}} b_{de}^c b_{ac}^a X_a X^d X_f \otimes X_b X^e X^c \in \mathcal{U}(\mathfrak{g})^{\otimes 2}$$



**Forbidden Theorem [EK, Ha, ?].** There exists a homomorphic expansion Z for vTT.

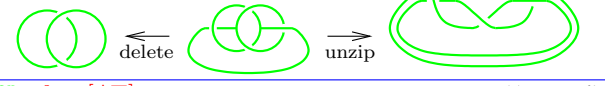
**Why Forbidden (to me)?**

- Minor statement details may be off.
- No fully written proof.
- I don't understand the proof.
- There isn't yet a knot-theoretic view of the proof, like there is in the w-case.



**Why Should We Care?**

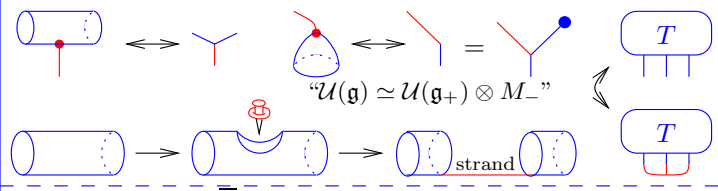
- A gateway into the forbidden territory of "quantum groups".
- Abstractly more pleasing: We study the things, and not just their representations.
- $\mathcal{A}^v$  is sometimes easier than  $\mathcal{A}^u$ : Alexander, say, arises easily from the 2D Lie algebra<sup>4</sup>.
- Potentially,  $\mathcal{A}^v$  has many more "internal quotients" than there are Lie bialgebras. What are they and what are the corresponding theories?
- My old<sup>5</sup> Algebraic Knot Theory dream:



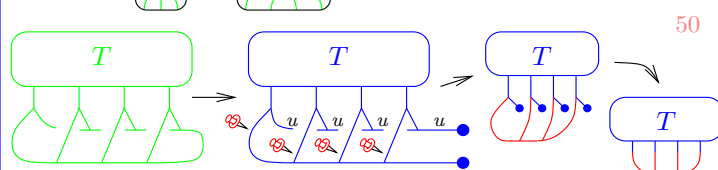
$V \rightarrow \Phi$ -loop after [AT]. "cut and cap" is well-defined(!) on  $\mathcal{K}^u$



$\Phi \rightarrow V$  after [AET]. In  $\mathcal{K}^w$  allow tubes and strand-vertices, allow "punctures", yet allow no "tangles".



The generators of  $\mathcal{K}^w$  can be written in terms of the generators of  $\mathcal{K}^u$  (i.e., given  $\Phi$ , can write a formula for V). With T any classical tangle, esp.  $\left( \begin{array}{c} \cap \\ \cup \end{array} \right)$  or  $\left( \begin{array}{c} \cup \\ \cap \end{array} \right)$ , consider the "sled"



Alexander is easy! In Chicago, [BN4] Many kinds of virtuals!

**Help Needed!**