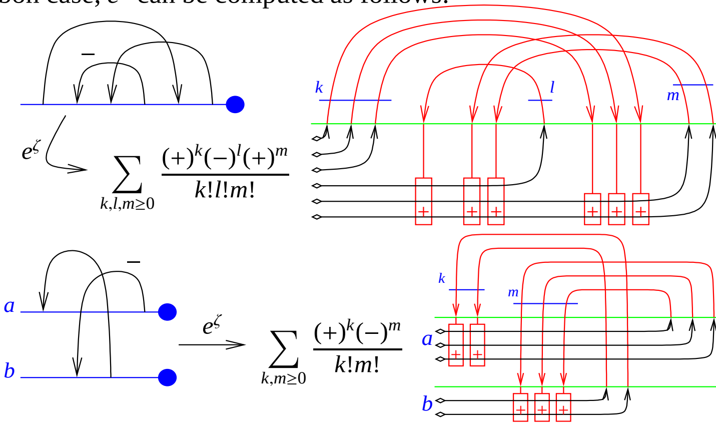
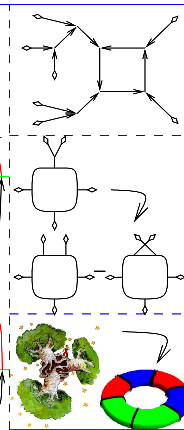
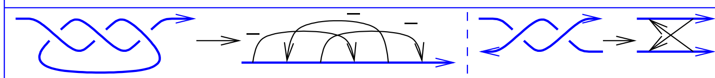
A Partial Reduction of BF Theory to Combinatorics, 2

Theorem 1 (with Cattaneo, Dalvit (credit, no blame)). In the ribbon case, e^ζ can be computed as follows:







Theorem 2. Using Gauss diagrams to represent knots and T-component pure tangles, the above formulas define an invariant in $CW(FL(T))\to CW(T)$, "cyclic words in T".
• Agrees with BN-Dancso [BND] and with [BN2]. • In-practice computable! • Vanishes on braids. • Extends to w. • Contains Alexander. • The "missing factor" in Levine's factorization [Le] (the rest of [Le] also fits, hence contains the MVA). • Related to / extends Farber's [Fa]? • Should be summed and categorified.

References.

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[BN2] D. Bar-Natan, *Balloons and Hoops and their Universal Finite Type Invariant, BF Theory, and an Ultimate Alexander Invariant,* http://www.math.toronto.edu/~drorbn/papers/KBH/, arXiv:1308.1721.

[BND] D. Bar-Natan and Z. Dancso, *Finite Type Invariants of W-Knotted Objects: From Alexander to Kashiwara and Vergne,* http://www.math.toronto.edu/~drorbn/papers/WKO/.

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[Da] E. Dalvit, http://science.unitn.it/~dalvit/.

[CR] A. S. Cattaneo and C. A. Rossi, *Wilson Surfaces and Higher Dimensional Knot Invariants,* Commun. in Math. Phys. **256-3** (2005) 513–537, arXiv:math-ph/0210037.

[Fa] M. Farber, *Noncommutative Rational Functions and Boundary Links,* Math. Ann. **293** (1992) 543–568.

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[Ro] D. Roseman, *Reidemeister-Type Moves for Surfaces in Four-Dimensional Space,* Knot Theory, Banach Center Publications **42** (1998) 347–380.

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Continuing Joost Slingerland...

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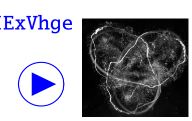
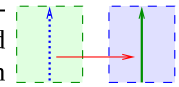
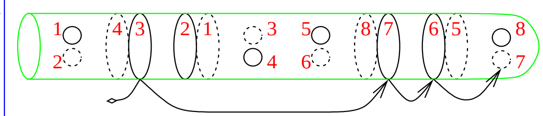


Sketch of Proof. In $4D$ axial gauge, only "drop down" red propagators, hence in the ribbon case, no M-trivalent vertices. S integrals are ± 1 iff "ground pieces" run on nested curves as below, and exponentials arise when several propagators compete for the same double curve. And then the combinatorics is obvious...

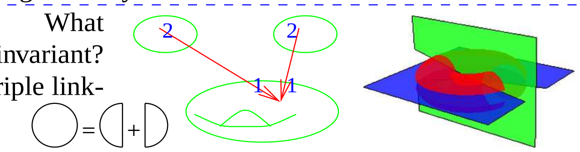




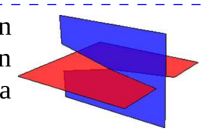
Musings

Chern-Simons. When the domain of BF is restricted to ribbon knots, and the target of Chern-Simons is restricted to trees and wheels, they agree. Why?

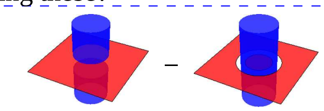
Is this all? What about the $\vee$-invariant? (the "true" triple linking number)



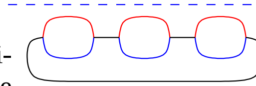
Gnots. In 3D, a generic immersion of S^1 is an embedding, a knot. In 4D, a generic immersion of a surface has finitely-many double points (a gnot?). Perhaps we should be studying these?



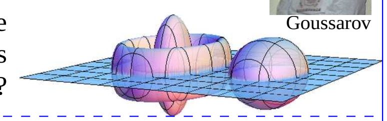
Finite type. What are finite-type invariants for 2-knots? What would be "chord diagrams"?



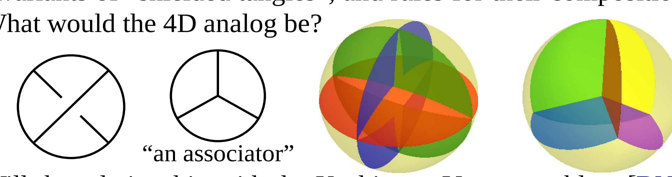
Bubble-wrap-finite-type. There's an alternative definition of finite type in 3D, due to Goussarov (see [BN1]). The obvious parallel in 4D involves "bubble wraps". Is it any good?




Goussarov

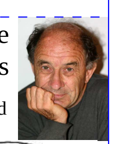


Shielded tangles. In 3D, one can't zoom in and compute "the Chern-Simons invariant of a tangle". Yet there are well-defined invariants of "shielded tangles", and rules for their compositions. What would the 4D analog be?

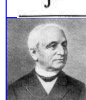

"an associator"

Will the relationship with the Kashiwara-Vergne problem [BND] necessarily arise here?

Plane curves. Shouldn't we understand integral / finite type invariants of plane curves, in the style of Arnold's J^+, J^-, and St [Ar], a bit better?


Arnold

	$a(\cdot)$	$a(\cdot)$	$a(\cdot)$						
St	1	0	0	0	0	1	2	3	...
J^+	0	2	0	0	0	−2	−4	−6	...
J^-	0	0	−2	−1	0	−3	−6	−9	...



"God created the knots, all else in topology is the work of mortals."

Leopold Kronecker (modified)

www.katlas.org

Video and more at http://www.math.toronto.edu/~drorbn/Talks/Vienna-1402/