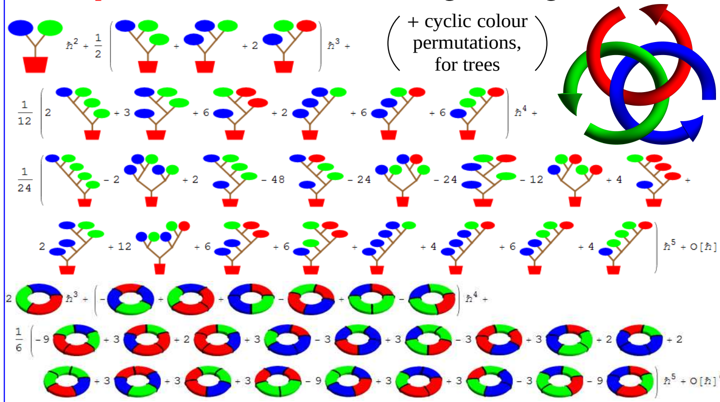
Work in Progress on Polynomial Time Knot Polynomials, B

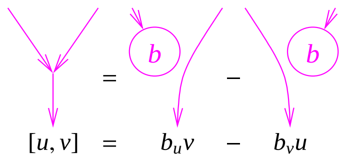
Theorem 2 [BND]. $\exists!$ a homomorphic expansion, aka a homomorphic universal finite type invariant Z^w of pure w-tangles. $z^w := \log Z^w$ takes values in $FL(S)^S \times CW(S)$.

z is computable. z of the Borromean tangle, to degree 5 [BN]:

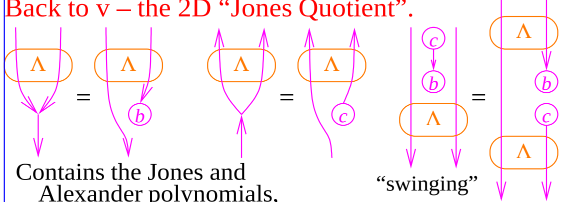


(I have a fancy free-Lie calculator!) (ωεβ/FLD) Nice, but too hard!

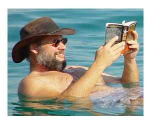
Proposition [BN]. Modulo all relations that universally hold for the 2D non-Abelian Lie algebra and after some changes-of-variable, z^w reduces to z_0.



Back to v – the 2D "Jones Quotient".



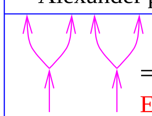
Contains the Jones and Alexander polynomials,



V. Jones

. . . yet still too hard!

The OneCo Quotient. Likely related to [ADO]



= 0, only one co-bracket is allowed.

Everything should work, and everything is being worked!

Definition. (Compare [BNS, BN]) A meta-monoid is a functor M: (finite sets, injections)→(sets) (think "$M(S)$ is quantum G^S", for G a group) along with natural operations $*: M(S_1)\times M(S_2) \to M(S_1 \sqcup S_2)$ whenever $S_1 \cap S_2 = \emptyset$ and $m^{ab}_c: M(S) \to M((S\setminus\{a,b\}) \sqcup \{c\})$ whenever $a \neq b \in S$ and $c \notin S\setminus\{a,b\}$, such that

The Abstract Context

$$\text{meta-associativity:}\quad m^{ab}_a /\!/ m^{ac}_a = m^{bc}_b /\!/ m^{ab}_a$$

$$\text{meta-locality:}\quad m^{ab}_c /\!/ m^{de}_f = m^{de}_f /\!/ m^{ab}_c$$

and, with $\epsilon_b = M(S \hookrightarrow S \sqcup \{b\})$,

$$\text{meta-unit:}\quad \epsilon_b /\!/ m^{ab}_a = Id = \epsilon_b /\!/ m^{ba}_a.$$

Claim. Pure virtual tangles PvT form a meta-monoid.

Theorem. $S \mapsto \Gamma_0(S)$ is a meta-monoid and $z_0: \mathit{PvT} \to \Gamma_0$ is a morphism of meta-monoids.

Strong Conviction. There exists an extension of Γ_0 to a bigger meta-monoid $\Gamma_{01}(S) = \Gamma_0(S)\times\Gamma_1(S)$, along with an extension of z_0 to $z_{01}: \mathit{PvT} \to \Gamma_{01}$, with

$$\Gamma_1(S) = V \oplus V^{\otimes 2} \oplus V^{\otimes 3} \oplus \mathcal{S}^2(V)^{\otimes 2} \qquad (\text{with } V := R_S\langle S\rangle).$$

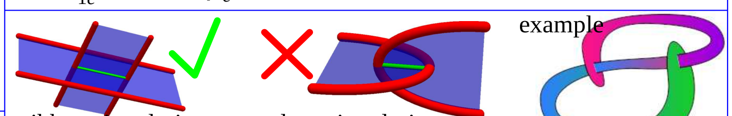
Furthermore, upon reducing to a single variable everything is polynomial size and polynomial time.

Furthermore, Γ_{01} is given using a "meta-2-cocycle ρ^{ab}_c over Γ_0": In addition to $m^{ab}_c \to m^{ab}_{0c}$, there are R_S-linear $m^{ab}_{1c}: \Gamma_1(S \sqcup \{a,b\}) \to \Gamma_1(S\sqcup\{c\})$, a meta-right-action $\alpha^{ab}: \Gamma_1(S)\times\Gamma_0(S) \to \Gamma_1(S)$ R_S-linear in the first variable, and a first order differential operator (over R_S) $\rho^{ab}_c: \Gamma_0(S\sqcup\{a,b\}) \to \Gamma_1(S\sqcup\{c\})$ such that

$$(\zeta_0,\zeta_1)/\!/m^{ab}_c = \left(\zeta_0/\!/m^{ab}_{0c}, (\zeta_1,\zeta_0)/\!/\alpha^{ab}/\!/m^{ab}_{1c} + \zeta_0/\!/\rho^{ab}_c\right)$$

What's done? The braid part, with still-ugly formulas.

What's missing? A lot of concept- and detail-sensitive work towards m^{ab}_{1c}, α^{ab}, and ρ^{ab}_c. The "ribbon element".



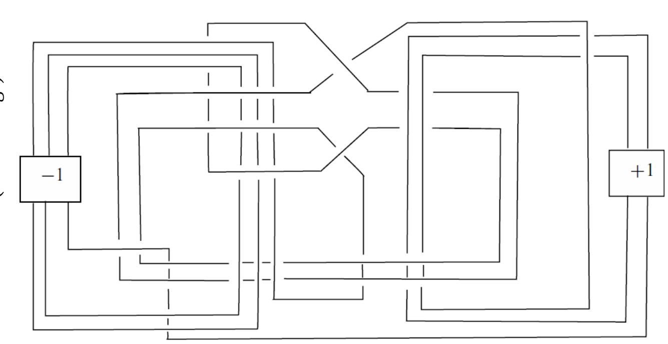
a ribbon singularity a clasp singularity

A bit about ribbon knots. A "ribbon knot" is a knot that can be presented as the boundary of a disk that has "ribbon singularities", but no "clasp singularities". A "slice knot" is a knot in $S^3 = \partial B^4$ which is the boundary of a non-singular disk in B^4. Every ribbon knots is clearly slice, yet,

Conjecture. Some slice knots are not ribbon.

Fox-Milnor. The Alexander polynomial of a ribbon knot is always of the form $A(t) = f(t)f(1/t)$. (also for slice)

[GST]: a slice knot that might not be ribbon (48 crossings).



References.

[ADO] Y. Akutsu, T. Deguchi, and T. Ohtsuki, *Invariants of Colored Links*, J. of Knot Theory and its Ramifications **1-2** (1992) 161–184.

[BN] D. Bar-Natan, *Balloons and Hoops and their Universal Finite Type Invariant, BF Theory, and an Ultimate Alexander Invariant*, ωεβ/KBH, arXiv:1308.1721.

[BND] D. Bar-Natan and Z. Dancso, *Finite Type Invariants of W-Knotted Objects I-II*, ωεβ/WKO1, ωεβ/WKO2, arXiv:1405.1956, arXiv:1405.1955.

[BNS] D. Bar-Natan and S. Selmani, *Meta-Monoids, Meta-Bicrossed Products, and the Alexander Polynomial*, J. of Knot Theory and its Ramifications **22-10** (2013), arXiv:1302.5689.

[CT] D. Cimasoni and V. Turaev, *A Lagrangian Representation of Tangles*, Topology **44** (2005) 747–767, arXiv:math.GT/0406269.

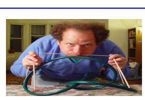
[En] B. Enriquez, *A Cohomological Construction of Quantization Functors of Lie Bialgebras*, Adv. in Math. **197-2** (2005) 430-479, arXiv:math/0212325.

[EK] P. Etingof and D. Kazhdan, *Quantization of Lie Bialgebras, I*, Selecta Mathematica **2** (1996) 1–41, arXiv:q-alg/9506005.

[GST] R. E. Gompf, M. Scharlemann, and A. Thompson, *Fibered Knots and Potential Counterexamples to the Property 2R and Slice-Ribbon Conjectures*, Geom. and Top. **14** (2010) 2305–2347, arXiv:1103.1601.

[KLW] P. Kirk, C. Livingston, and Z. Wang, *The Gassner Representation for String Links*, Comm. Cont. Math. **3** (2001) 87–136, arXiv:math/9806035.

[LD] J. Y. Le Dimet, *Enlacements d'Intervalles et Représentation de Gassner*, Comment. Math. Helv. **67** (1992) 306–315.



Help Needed!

I'm slow and feeble-minded.

"God created the knots, all else in topology is the work of mortals."

Leopold Kronecker (modified)

www.katlas.org

Video and more at http://www.math.toronto.edu/~drorbn/Talks/LesDiablerets-1508/