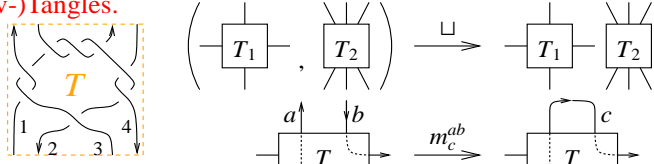




Abstract. The value of things is inversely correlated with their computational complexity. "Real time" machines, such as our brains, only run linear time algorithms, and there's still a lot we don't know. Anything we learn about things doable in linear time is truly valuable. Polynomial time we can in-practice run, even if we have to wait; these things are still valuable. Exponential time we can play with, but just a little, and exponential things must be beautiful or philosophically compelling to deserve attention. Values further diminish and the aesthetic-or-philosophical bar further rises as we go further slower, or un-computable, or ZFC-style intrinsically infinite, or large-cardinalish, or beyond.

I will explain some things I know about polynomial time knot polynomials and explain where there's more, within reach.

(v-)Tangles.



Why Tangles?

- Finitely presented. (meta-associativity: $m_a^{ab} // m_a^{ca} = m_b^{bc} // m_a^{ab}$)
 - Divide and conquer proofs and computations.
 - "Algebraic Knot Theory": If K is ribbon, $z(K) \in \{cl_2(\zeta) : cl_1(\zeta) = 1\}$.
- (Genus and crossing number are also definable properties). $U \in \mathcal{T}_n$, $K \in \mathcal{T}_1$. cl_1 : trivial, cl_2 : ribbon. Faster is better, leaner is meaner!

Theorem 1. $\exists!$ an invariant z_0 : {pure framed S -component tangles} $\rightarrow \Gamma_0(S) := R \times M_{S \times S}(R)$, where $R = R_S = \mathbb{Z}((T_a)_{a \in S})$ is the ring of rational functions in S variables, intertwining

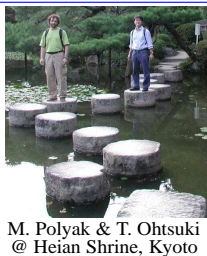
$$\begin{pmatrix} \omega_1 & S_1 \\ S_1 & A_1 \end{pmatrix}, \begin{pmatrix} \omega_2 & S_2 \\ S_2 & A_2 \end{pmatrix} \xrightarrow{\sqcup} \begin{pmatrix} \omega_1 \omega_2 & S_1 & S_2 \\ S_1 & A_1 & 0 \\ S_2 & 0 & A_2 \end{pmatrix}$$

$$\begin{pmatrix} \omega & a & b & S \\ a & \alpha & \beta & \theta \\ b & \gamma & \delta & \epsilon \\ S & \phi & \psi & \Xi \end{pmatrix} \xrightarrow{m_c^{ab}} \begin{pmatrix} \mu\omega & c & S \\ c & \gamma + \alpha\delta/\mu & \epsilon + \delta\theta/\mu \\ S & \phi + \alpha\psi/\mu & \Xi + \psi\theta/\mu \end{pmatrix}$$

and satisfying $(|a; a \nearrow b, b \nearrow a) \xrightarrow{z_0} \begin{pmatrix} 1 & a & b \\ a & 1 & 1 - T_a^{\pm 1} \\ b & 0 & T_a^{\pm 1} \end{pmatrix}$.

In Addition • The matrix part is just a stitching formula for Burau/Gassner [LD, KLV, CT].

- $K \mapsto \omega$ is Alexander, mod units.
- $L \mapsto (\omega, A) \mapsto \omega \det(A - I)/(1 - T')$ is the MVA, mod units.
- The fastest Alexander algorithm I know.
- There are also formulas for strand deletion, reversal, and doubling.
- Every step along the computation is the invariant of something.
- Extends to and more naturally defined on v/w-tangles.
- Fits in one column, including propaganda & implementation.



Implementation key idea:

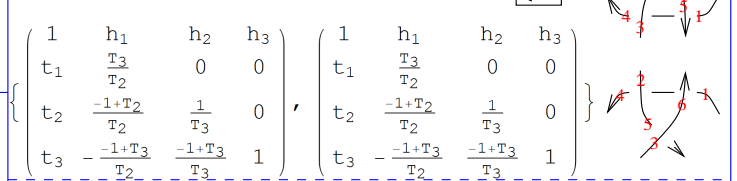
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F := F[\omega_1, \lambda_1] F[\omega_2, \lambda_2] := F[\omega_1 * \omega_2, \lambda_1 * \lambda_2];
m_{a,b} \to c := Module[\alpha, \beta, \gamma, \delta, \theta, \epsilon, \phi, \psi, \Xi, \mu];
[ \alpha \beta \theta ] := [ \partial_{\alpha, \beta, \gamma} \partial_{\alpha, \beta, \gamma} \partial_{\alpha, \beta, \gamma} ];
[ \gamma \delta \epsilon ] := [ \partial_{\gamma, \delta, \epsilon} \partial_{\gamma, \delta, \epsilon} \partial_{\gamma, \delta, \epsilon} ];
[ \phi \psi \Xi ] := [ \partial_{\phi, \psi, \Xi} \partial_{\phi, \psi, \Xi} \partial_{\phi, \psi, \Xi} ];
Gamma[\mu := 1 - \beta, \omega, {t_c}, 1] := (\gamma + \alpha \delta / \mu \epsilon + \delta \theta / \mu) . (t | h)_{a,b} \to 0;
/. {T_a \to T_c, T_b \to T_c} // RCollect];
FP_{a,b} := Gamma[1, {t_a, t_b} . (1 - T_a) . {h_a, h_b}];
RM_{a,b} := RP_{ab} /. T_a \to 1 / T_a;
  
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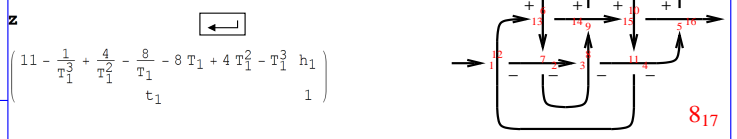
Meta-Associativity $\xi = \Gamma[\omega, \{t_1, t_2, t_3, t_s\}] \cdot \{h_1, h_2, h_3, h_s\}$; **Runs.**

$(\xi // m_{12 \rightarrow 1} // m_{13 \rightarrow 1}) = (\xi // m_{23 \rightarrow 2} // m_{12 \rightarrow 1})$

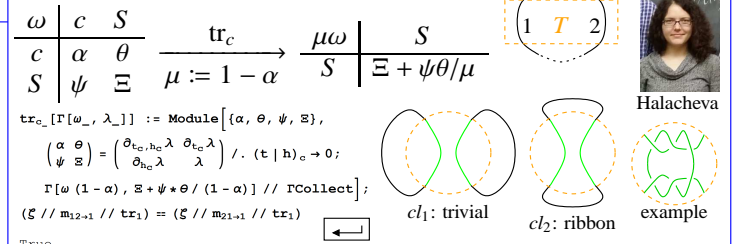
True **R3** ... divide and conquer!
 $\{Rm_{51} Rm_{62} Rp_{34} // m_{14 \rightarrow 1} // m_{25 \rightarrow 2} // m_{36 \rightarrow 3},$
 $Rp_{61} Rm_{24} Rm_{35} // m_{14 \rightarrow 1} // m_{25 \rightarrow 2} // m_{36 \rightarrow 3}\}$



$z = Rm_{12,1} Rm_{27} Rm_{83} Rm_{4,11} Rp_{16,5} Rp_{6,13} Rp_{14,9} Rp_{10,15}$;
Do $[z = z // m_{1k \rightarrow 1}, \{k, 2, 16\}]$;
 z

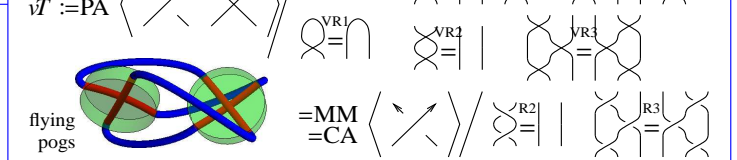


Closed Components. The Halacheva trace tr_c satisfies $m_c^{ab} // tr_c = m_c^{ba} // tr_c$ and computes the MVA for all links in the atlas, but its domain is not understood:



Weaknesses. • m_c^{ab} and tr_c are non-linear. • The product ωA is always Laurent, but my current proof takes induction with exponentially many conditions. • I still don't understand tr_c , "unitarity", the algebra for ribbon knots. **Where does it come from?**

v-Tangles.



Let $\mathcal{I} := \langle \times, - \times \rangle$. Then $\mathcal{A}^v := \prod I^n / I^{n+1} = \text{"universal } \mathcal{U}(Dg)^{\otimes S} =$
 $\langle \times, - \times \rangle \rightarrow \mathcal{A}^v$ (Also IHX)
Fine print: No sources no sinks, AS vertices, internally acyclic, deg = (#vertices)/2.

Likely Theorem. [EK, En] There exists a homomorphic expansion (universal finite type invariant) $Z: vT \rightarrow \mathcal{A}^v$. (issues suppressed)

Too hard! Let's look for "meta-monoid" quotients.

