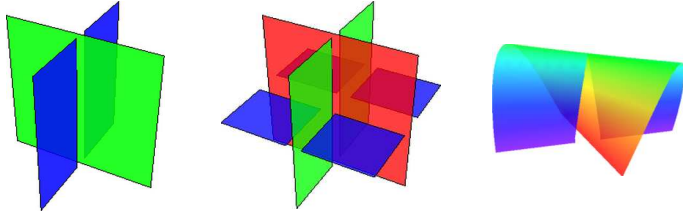
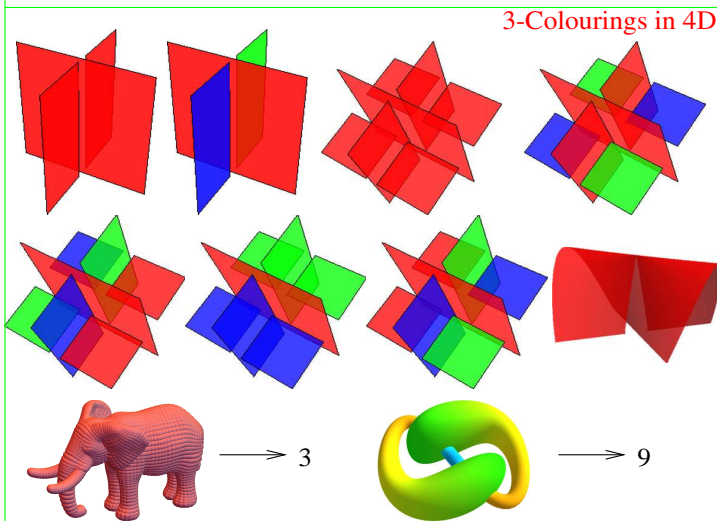
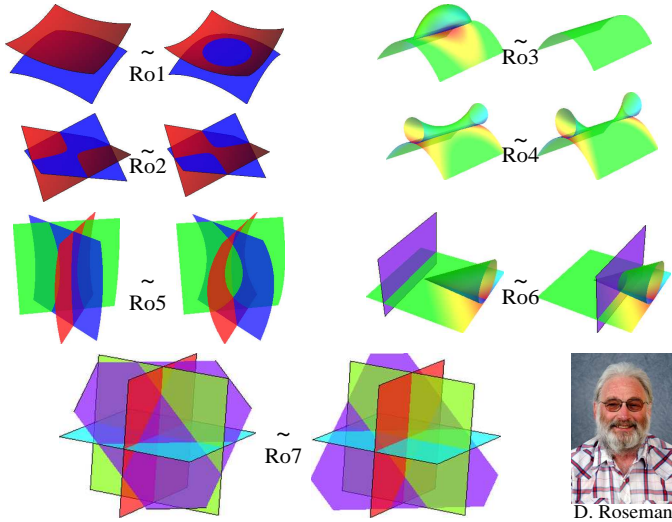


Theorem. Every 2-knot can be represented by a “broken surface diagram” made of the following basic ingredients,



... and any two representations of the same knot differ by a sequence of the following “Roseman moves”:



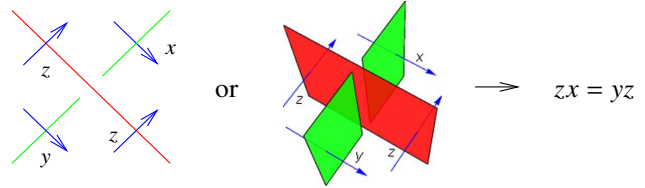
A Knot Table

There are many more!

Unknot	3_1	4_1	5_1	5_2
6_1	6_2	6_3	7_1	7_2
7_3	7_4	7_5	7_6	7_7

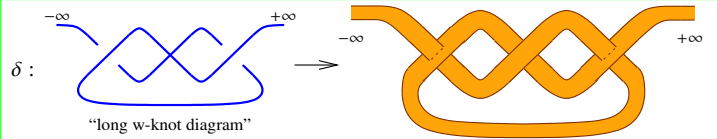
ω/KT

A Stronger Invariant. There is an assignment of groups to knots / 2-knots as follows. Put an arrow “under” every un-broken curve / surface in a broken curve / surface diagram and label it with the name of a group generator. Then mod out by relations as below.

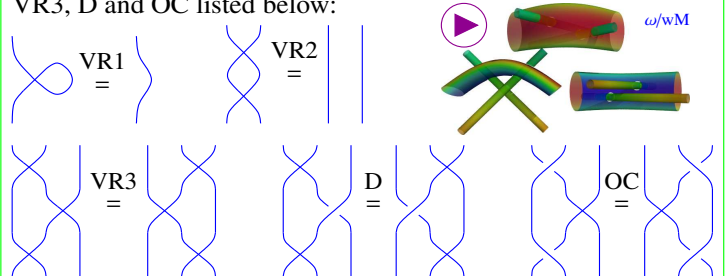


Facts. The resulting “Fundamental group” $\pi_1(K)$ of a knot / 2-knot K is a very strong but not very computable invariant of K . Though it has computable projections; e.g., for any finite G , count the homomorphisms from $\pi_1(K)$ to G .

Exercise. Show that $|\text{Hom}(\pi_1(K) \rightarrow S_3)| = \lambda(K) + 3$.



Satoh's Conjecture. (Satoh, *Virtual Knot Presentations of Ribbon Torus-Knots*, J. Knot Theory and its Ramifications **9** (2000) 531–542). Two long w-knot diagrams represent via the map δ the same simple long 2D knotted tube in 4D iff they differ by a sequence of R-moves as above and the “w-moves” VR1–VR3, D and OC listed below:



Some knot theory books.

- Colin C. Adams, *The Knot Book, an Elementary Introduction to the Mathematical Theory of Knots*, American Mathematical Society, 2004.
- Meike Akveld and Andrew Jobbings, *Knobs Unravalled, from Strings to Mathematics*, Arbelos 2011.
- J. Scott Carter and Masahico Saito, *Knotted Surfaces and Their Diagrams*, American Mathematical Society, 1997.
- Peter Cromwell, *Knobs and Links*, Cambridge University Press, 2004.
- W.B. Raymond Lickorish, *An Introduction to Knot Theory*, Springer 1997.

