Loading KnotTheory`

```
<< KnotTheory`
```

Loading KnotTheory` version
of September 6, 2014, 13:37:37.2841.
Read more at http://katlas.org/wiki/KnotTheory.

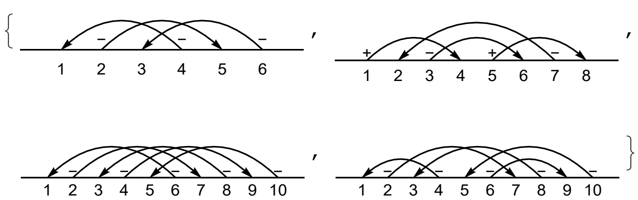
Gauss Diagram Utilities

```
GD[g_GD] := g;
GD[L_] := GD @@ PD[L] /.
   X[i_, j_, k_, l_] :> If[PositiveQ@X[i, j, k, l],
     Ap_{l,i}, Am_{j,i}];
Draw[g_GD] := Module[{n = Max@Cases[g, _Integer, ∞]},
  Graphics[{
    Line[{{0, 0}, {n + 1, 0}}],
    List @@ g /. (ah_)_{i_,j_} :> {
       Arrow[BezierCurve[{{i, 0}, {i + j, Abs[j - i]} / 2,
          {j, 0}}]],
       Text[ah /. {Ap → "+", Am → "-"}, {i, 0.3}]},
    Table[Text[i, {i, -0.5}], {i, n}]}]]
```

Some Gauss Diagrams

```
Draw /@ GD /@ AllKnots@{3, 5}
```

KnotTheory::loading : Loading precomputed data in PD4Knots`.



Some Gauss Diagrams, 2

```
GD /@ AllKnots@{3, 5}
```

{GD[$Am_{4,1}$, $Am_{6,3}$, $Am_{2,5}$], GD[$Ap_{1,4}$, $Ap_{5,8}$, $Am_{3,6}$, $Am_{7,2}$],
GD[$Am_{6,1}$, $Am_{8,3}$, $Am_{10,5}$, $Am_{2,7}$, $Am_{4,9}$],
GD[$Am_{4,1}$, $Am_{8,3}$, $Am_{10,5}$, $Am_{6,9}$, $Am_{2,7}$]}

V_2 Definition

```
CF[g_GD] := Sort[
   g /. Thread[Sort@Cases[g, _Integer, ∞] →
      Range[2 Length[g]]]];
PV[F_GD, g_GD] /; Length[F] > Length[g] := 0;
PV[F_GD, g_GD] /; Length[F] < Length[g] := Sum[
   PV[F, γ], {γ, Subsets[g, {Length[F]}]}];
PV[F_GD, g_GD] /; Length[F] == Length[g] := If[
   CF[F] === CF[g /. Ap | Am → A], (-1)^Count[g, Am__], 0];
V2[g_] := V2[g] = PV[GD[A_{3,1}, A_{2,4}], GD[g]];
```

Computing V_2

```
Format[Knot[n_, k_]] := n_k;
Table[K → V2[K], {K, AllKnots@{3, 7}}]
```

{$3_1 \to 1$, $4_1 \to -1$, $5_1 \to 3$, $5_2 \to 2$, $6_1 \to -2$, $6_2 \to -1$, $6_3 \to 1$,
$7_1 \to 6$, $7_2 \to 3$, $7_3 \to 5$, $7_4 \to 4$, $7_5 \to 4$, $7_6 \to 1$, $7_7 \to -1$}

V_3 Definition

```
PV[F1_ + F2_, g_] := PV[F1, g] + PV[F2, g];
PV[c_ * F_GD, g_] := c PV[F, g];
ρ_k_[g_] := g /. i_Integer :> Mod[i - k, 2 Length@g, 1];
F3 = Sum[(3 ρ_k@GD[A_{1,5}, A_{4,2}, A_{6,3}] + 2 ρ_k@GD[A_{1,4}, A_{5,2}, A_{3,6}]), {k, 0, 5}];
V3[K_] := V3[K] = PV[F3, GD@K] / 6;
```

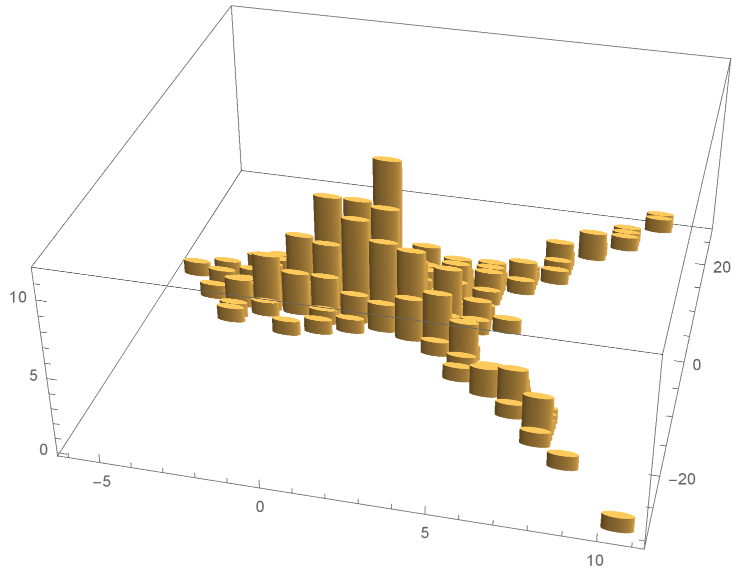
Computing V_3

```
Table[K → V3[K], {K, AllKnots@{3, 7}}]
```

{$3_1 \to -1$, $4_1 \to 0$, $5_1 \to -5$, $5_2 \to -3$, $6_1 \to 1$, $6_2 \to 1$, $6_3 \to 0$,
$7_1 \to -14$, $7_2 \to -6$, $7_3 \to 11$, $7_4 \to 8$, $7_5 \to -8$, $7_6 \to -2$, $7_7 \to -1$}

Willerton's Fish

```
Histogram3D[
 Table[{V2[K], V3[K]}, {K, AllKnots@{3, 10}}],
 {1}]
```



Gassner Utilities

```
G[λ_]_{a_,b_} := ∂_{t_a,h_b} λ;
G /: Factor[G[λ_]] :=
  G[Collect[λ, h_, Collect[#, t_, Factor] &]];
Format@γ_G := Module[{S = Union@Cases[γ, (h | t)_a_ :> a, ∞]},
   Table[γ_{a,b}, {a, S}, {b, S}] // MatrixForm];
```

The Gassner Program

```
G /: G[λ1_] G[λ2_] := G[λ1 + λ2];
m_{a_,b_→c_}[G[λ_]] := Module[{α, β, γ, δ, θ, ϵ, ϕ, ψ, Ξ, μ},
   {{α, β, θ}, {γ, δ, ϵ}, {ϕ, ψ, Ξ}} =
     {{∂_{t_a,h_a} λ, ∂_{t_a,h_b} λ, ∂_{t_a} λ},
      {∂_{t_b,h_a} λ, ∂_{t_b,h_b} λ, ∂_{t_b} λ},
      {∂_{h_a} λ, ∂_{h_b} λ, λ}} /. (t | h)_{a|b} → 0;
   μ = 1 - β;
   G[Tr[{{t_c}, {1}}^T . {{γ + α δ / μ, ϵ + δ θ / μ}, {ϕ + α ψ / μ, Ξ + ψ θ / μ}} . {{h_c}, {1}}]] /. T_{a|b} → T_c //
    Factor];
Rp_{a_,b_} := G[Tr[{{t_a}, {t_b}}^T . {{1, 1 - T_a}, {0, T_a}} . {{h_a}, {h_b}}]];
Rm_{a_,b_} := Rp_{a,b} /. T_a → 1 / T_a;
```

The Gauss-Gassner-Program

```
GG[g_GD, k_, F_, BB_] :=
  Module[{n = 2 Length@g + Length@BB, y, cuts, rr, γ0, γ},
   γ0 = G[t_{n+1} h_{n+1}] Times @@ g /. {Ap → Rp, Am → Rm};
   γ0 *= G[Sum[β_{a,b} t_a h_b, {a, BB}, {b, BB}]];
   Sum[ γ = γ0;
    cuts = Cases[y, _Integer, ∞] ⋃ {n + 1};
    rr = Thread[cuts → Range[Length@cuts]];
    Do[If[! MemberQ[cuts, j], γ = γ // m_{j,j+1→j+1}], {j, n}];
    F[y /. rr, γ /. (v_)_a_ :> v_{a/.rr}],
    (*over*) {y, Subsets[List @@ g, k]}]];
GG[g_GD, k_, F_] := GG[g, k, F, {}];
```