Abstract. I will describe a semi-rigorous reduction of perturbative BF theory (Cattaneo-Rossi [CR]) to computable combinatorics, in the case of ribbon 2-links. Also, I will explain how and why my approach may or may not work in the non-ribbon case. Weak this result is, and at least partially already known (Watanabe [Wa]). Yet in the ribbon case, the resulting invariant is a universal finite type invariant, a gadget that significantly generalizes and clarifies the Alexander polynomial and that is closely related to the Kashiwara-Vergne problem. I cannot rule out the possibility that the corresponding gadget in the non-ribbon case will be as interesting.

BF Following [CR]. $A \in \Omega^1(M = \mathbb{R}^4, g), B \in \Omega^2(M, g^*)$, 

$S(A, B) := \int_M \langle B, F_A \rangle$.

With $\kappa: (S = \mathbb{R}^2) \to M, \beta \in \Omega^0(S, g), \alpha \in \Omega^1(S, g^*)$, set 

$O(A, B, \kappa) := \int DBDA \exp \left( \frac{i}{\hbar} \int_S (\beta, \Delta_A \alpha + \kappa^* B) \right)$.

Decker Sets (“2D Gauss Codes”).

“a double curve”

“a triple point”

“a branch point”

Some Examples.

A 4D knot by Carter and Saito [CS]

A 4D knot by Dalvit [Da]

A 2-link


A Partial Reduction of BF Theory to Combinatorics, 1

The BF Feynman Rules. For an edge $e$, let $\Phi_e$ be its direction, in $S^3$ or $S^1$. Let $\omega_1$ and $\omega_2$ be volume forms on $S^3$ and $S^1$. Then for a 2-link $(k_i) \in \mathbb{T}$,

$\zeta = \log \sum_{\text{diagrams}_D} \frac{[D]}{\text{Aut}(D)} \int_S \int_{\mathbb{R}^3} \int_{\mathbb{R}^2} \int_{\mathbb{R}^3} \int_{\mathbb{R}^3} \prod_{S\text{-vertices}} \Phi_e \omega_1 \prod_{M\text{-vertices}} \Phi_e \omega_2$

is an invariant in $\text{CW}(FL(T)) \to \text{CW}(T)/\sim$, “symmetrized cyclic words in $T$”.

A BF Feynman Diagram.