

Demo Programs for 0-Co.

ωεβ/Demo

$$R_{\theta, i, j}^+ := \mathbb{E} [b_i c_j + b_i^{-1} (e^{b_i} - 1) u_i w_j];$$

$$R_{\theta, i, j}^- := \mathbb{E} [-b_i c_j + b_i^{-1} (e^{-b_i} - 1) u_i w_j];$$

The R-matrices

CF[ω<sub>-</sub>. E[Q<sub>-</sub>]] := Simplify[ω E[Simplify[Q]]];  
 E /: E[Q1<sub>-</sub>] E[Q2<sub>-</sub>] := CF@E[Q1 + Q2];  
 ω1<sub>-</sub>. E[Q1<sub>-</sub>] ≡ ω2<sub>-</sub>. E[Q2<sub>-</sub>] := Simplify[ω1 == ω2 ∧ Q1 == Q2];

Utilities

N<sub>(x:w|u)</sub><sub>i</sub> c<sub>j</sub> →<sub>k</sub> [ω<sub>-</sub>. E[Q<sub>-</sub>]] := CF [ Normal Ordering Operators  
 ω E[e<sup>α</sup> x<sub>k</sub> + γ c<sub>k</sub> + (Q / . c<sub>j</sub> | x<sub>i</sub> → θ)] / . {γ → ∂<sub>c<sub>j</sub></sub> Q, α → ∂<sub>x<sub>i</sub></sub> Q};  
 N<sub>w</sub><sub>i</sub> u<sub>j</sub> →<sub>k</sub> [ω<sub>-</sub>. E[Q<sub>-</sub>]] := CF [  
 v ω E[-b<sub>k</sub> v α β + v β u<sub>k</sub> + v α w<sub>k</sub> + v δ u<sub>k</sub> w<sub>k</sub> + (Q / . w<sub>i</sub> | u<sub>j</sub> → θ)] / .  
 v → (1 + b<sub>k</sub> δ)<sup>-1</sup> / .  
 {α → ∂<sub>w<sub>i</sub></sub> Q / . u<sub>j</sub> → θ, β → ∂<sub>u<sub>j</sub></sub> Q / . w<sub>i</sub> → θ, δ → ∂<sub>w<sub>i</sub>, u<sub>j</sub></sub> Q}];

Stitching

m<sub>i, j</sub> →<sub>k</sub> [Z<sub>-</sub>] := Module[{X, Z},  
 CF[(Z // N<sub>w<sub>i</sub></sub> u<sub>j</sub> →<sub>k</sub> // N<sub>c<sub>i</sub></sub> u<sub>x</sub> →<sub>x</sub> // N<sub>w<sub>x</sub></sub> c<sub>j</sub> →<sub>x</sub>) / . Z<sub>-i</sub>|j|x → Z<sub>k</sub>]]

T<sub>0</sub> = R<sub>0,5,1</sub><sup>+</sup> R<sub>0,2,4</sub><sup>+</sup> R<sub>0,3,6</sub><sup>+</sup> Some calculations for T<sub>0</sub>

$$\mathbb{E} \left[ b_5 c_1 + b_2 c_4 - b_3 c_6 + \frac{(-1+e^{b_5}) u_5 w_1}{b_5} + \frac{(-1+e^{b_2}) u_2 w_4}{b_2} + \frac{(-1+e^{-b_3}) u_3 w_6}{b_3} \right]$$

T<sub>0</sub> // m<sub>1,2</sub> →<sub>1</sub> // m<sub>3,4</sub> →<sub>3</sub> // m<sub>3,5</sub> →<sub>3</sub> // m<sub>3,6</sub> →<sub>3</sub>

$$\frac{1}{1 - (-1+e^{b_1}) (-1+e^{b_3})} \mathbb{E} \left[ b_3 c_1 + b_1 c_3 - b_3 c_3 + \frac{e^{b_3} (-1+e^{b_1}) (-1+e^{b_3}) u_1 w_1}{(-e^{b_1} - e^{b_3} + e^{b_1+b_3}) b_1} - \frac{e^{b_1} (-1+e^{b_3}) u_3 w_1}{(-1 + (-1+e^{b_1}) (-1+e^{b_3})) b_3} - \frac{e^{-b_3} (-1+e^{b_1}) u_3 w_3}{b_3} - \frac{e^{-b_3} (-1+e^{b_1}) (-e^{b_3} b_3 u_1 + e^{b_1} (-1+e^{b_3}) b_1 u_3) w_3}{b_1 (b_3 - (-1+e^{b_1}) (-1+e^{b_3}) b_3)} \right]$$

Verifying meta-associativity

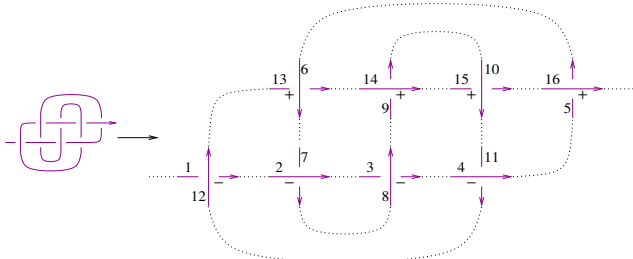
Q0 = E[Sum[f<sub>i</sub> c<sub>i</sub>, {i, 3}] + Sum[f<sub>i,j</sub> u<sub>i</sub> w<sub>j</sub>, {i, 3}, {j, 3}]]  
 E[C<sub>1</sub> f<sub>1</sub> + C<sub>2</sub> f<sub>2</sub> + C<sub>3</sub> f<sub>3</sub> + u<sub>1</sub> w<sub>1</sub> f<sub>1,1</sub> + u<sub>1</sub> w<sub>2</sub> f<sub>1,2</sub> + u<sub>1</sub> w<sub>3</sub> f<sub>1,3</sub> + u<sub>2</sub> w<sub>1</sub> f<sub>2,1</sub> +  
 u<sub>2</sub> w<sub>2</sub> f<sub>2,2</sub> + u<sub>2</sub> w<sub>3</sub> f<sub>2,3</sub> + u<sub>3</sub> w<sub>1</sub> f<sub>3,1</sub> + u<sub>3</sub> w<sub>2</sub> f<sub>3,2</sub> + u<sub>3</sub> w<sub>3</sub> f<sub>3,3}]]  
 (Q0 // m<sub>1,2</sub> →<sub>1</sub> // m<sub>1,3</sub> →<sub>1</sub>) ≡ (Q0 // m<sub>2,3</sub> →<sub>2</sub> // m<sub>1,2</sub> →<sub>1</sub>)  
 True</sub>

t1 = R<sub>0,1,2</sub><sup>+</sup> R<sub>0,3,4</sub><sup>+</sup> R<sub>0,5,6</sub><sup>+</sup> // m<sub>3,5</sub> →<sub>x</sub> // m<sub>1,6</sub> →<sub>y</sub> // m<sub>2,4</sub> →<sub>z</sub> Testing R3

$$\mathbb{E} [b_x c_y + b_x c_z + b_y c_z + \frac{e^{b_x} (-1+e^{b_y}) u_y w_z}{b_y} + \frac{(-1+e^{b_x}) u_x (w_y + w_z)}{b_x}]$$

t1 ≡ (R<sub>0,1,2</sub><sup>+</sup> R<sub>0,3,4</sub><sup>+</sup> R<sub>0,5,6</sub><sup>+</sup> // m<sub>1,3</sub> →<sub>x</sub> // m<sub>2,5</sub> →<sub>y</sub> // m<sub>4,6</sub> →<sub>z</sub>)

True



z1 = R<sub>0,12,1</sub><sup>+</sup> R<sub>0,2,7</sub><sup>+</sup> R<sub>0,8,3</sub><sup>+</sup> R<sub>0,4,11</sub><sup>+</sup> R<sub>0,16,5</sub><sup>+</sup> R<sub>0,6,13</sub><sup>+</sup> R<sub>0,14,9</sub><sup>+</sup> R<sub>0,10,15</sub><sup>+</sup>

Do[z1 = (z1 // m<sub>1,n</sub> →<sub>1</sub>) / . b<sub>-</sub> → b, {n, 2, 16}];  
 {CF@z1, KnotData[{8, 17}, "AlexanderPolynomial"] [t]}

$$\left\{ -\frac{e^{3b} E[0]}{1-4e^{b,8} e^{2b,11} e^{3b,8} e^{4b,4} e^{5b,6} e^b}, 11 - \frac{1}{t^3} + \frac{4}{t^2} - \frac{8}{t} - 8t + 4t^2 - t^3 \right\}$$

Demo Programs for 1-Co.

ωεβ/Demo

$$\Delta[k_-] := ((t_k - 1) (2 (\alpha \beta + \delta \mu)^2 - \alpha^2 \beta^2) - 4 v_k c_k w_k \delta^2 \mu^2 - \delta (1 + \mu) (w_k^2 \alpha^2 + v_k^2 \beta^2) - v_k^2 w_k^2 \delta^3 (1 + 3 \mu) - 2 (\alpha \beta + 2 \delta \mu + v_k w_k \delta^2 (1 + 2 \mu) + 2 c_k \delta \mu^2) (w_k \alpha + v_k \beta) - 4 (c_k \mu^2 + v_k w_k \delta (1 + \mu)) (\alpha \beta + \delta \mu) (1 + t_k) / 4; \text{The } \text{\u039b\u039f\u039f\u0399}$$

$$R_{i,j}^+ := \mathbb{E} [1, \text{Log}[t_i] c_j, v_i w_j, v_i c_i w_j + c_i c_j + v_i^2 w_j^2 / 4];$$

$$R_{i,j}^- := \mathbb{E} [1, -\text{Log}[t_i] c_j, -t_i^{-1} v_i w_j, t_i^{-1} v_i c_j w_j - c_i c_j - t_i^{-2} v_i^2 w_j^2 / 4];$$

$$(ur_{i-} := \mathbb{E} [t_i^{-1/2}, \theta, \theta, c_i t_i^2]; nr_{i-} := \mathbb{E} [t_i^{1/2}, \theta, \theta, -c_i t_i^2];)$$

The Generators

Differential Polynomials

DP<sub>x<sub>-</sub> →<sub>α</sub>, y<sub>-</sub> →<sub>β</sub></sub> [P<sub>-</sub>] [f<sub>-</sub>] := (\* means P[∂<sub>α</sub>, ∂<sub>β</sub>] [f] \*)  
 Total[CoefficientRules[P, {x, y}] / .  
 ({m<sub>-</sub>, n<sub>-</sub>} → c<sub>-</sub>) ⇒ c D[f, {α, m}, {β, n}]]

Utilities

CF[E<sub>-</sub>E] := Expand /@ Together /@ E;  
 E /: E[ω1<sub>-</sub>, L1<sub>-</sub>, Q1<sub>-</sub>, P1<sub>-</sub>] E[ω2<sub>-</sub>, L2<sub>-</sub>, Q2<sub>-</sub>, P2<sub>-</sub>] :=  
 CF@E[ω1 ω2, L1 + L2, ω2 Q1 + ω1 Q2, ω2<sup>4</sup> P1 + ω1<sup>4</sup> P2];

Normal Ordering Operators

N<sub>c<sub>j</sub></sub> (x:v|w)<sub>i</sub> →<sub>k</sub> [E[ω<sub>-</sub>, L<sub>-</sub>, Q<sub>-</sub>, P<sub>-</sub>]] := With[{q = e<sup>γ</sup> β x<sub>k</sub> + γ c<sub>k</sub>}, CF [  
 E[ω, γ c<sub>k</sub> + (L / . c<sub>j</sub> → θ), ω e<sup>γ</sup> β x<sub>k</sub> + (Q / . x<sub>i</sub> → θ),  
 e<sup>-q</sup> DP<sub>c<sub>j</sub> →<sub>γ</sub>, x<sub>i</sub> →<sub>β</sub></sub> [P] [e<sup>q</sup>]] / . {γ → ∂<sub>c<sub>j</sub></sub> L, β → ω<sup>-1</sup> ∂<sub>x<sub>i</sub></sub> Q}]]];

N<sub>w<sub>i</sub></sub> v<sub>j</sub> →<sub>k</sub> [E[ω<sub>-</sub>, L<sub>-</sub>, Q<sub>-</sub>, P<sub>-</sub>]] :=  
 With[{q = ((1 - t<sub>k</sub>) α β + β v<sub>k</sub> + α w<sub>k</sub> + δ v<sub>k</sub> w<sub>k}) / μ}, CF [  
 E[μ ω, L, μ ω q + μ (Q / . w<sub>i</sub> | v<sub>j</sub> → θ),  
 μ<sup>4</sup> e<sup>-q</sup> DP<sub>w<sub>i</sub> →<sub>α</sub>, v<sub>j</sub> →<sub>β</sub></sub> [P] [e<sup>q</sup>]] / . μ → 1 + (t<sub>k</sub> - 1) δ / .  
 {α → ω<sup>-1</sup> (∂<sub>w<sub>i</sub></sub> Q / . v<sub>j</sub> → θ), β → ω<sup>-1</sup> (∂<sub>v<sub>j</sub></sub> Q / . w<sub>i</sub> → θ),  
 δ → ω<sup>-1</sup> ∂<sub>w<sub>i</sub>, v<sub>j</sub></sub> Q}]]];</sub>

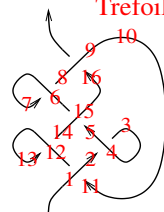
Stitching

m<sub>i, j</sub> →<sub>k</sub> [Z<sub>-</sub>E] := Module[{X, Z},  
 CF[(Z // N<sub>w<sub>i</sub></sub> v<sub>j</sub> →<sub>k</sub> // N<sub>c<sub>i</sub></sub> v<sub>x</sub> →<sub>x</sub> // N<sub>w<sub>x</sub></sub> c<sub>j</sub> →<sub>x</sub>) / . Z<sub>-i</sub>|j|x → Z<sub>k</sub>]]

z2 = R<sub>1,11</sub><sup>+</sup> R<sub>4,2</sub><sup>+</sup> nr<sub>3</sub> R<sub>15,5</sub><sup>+</sup> R<sub>6,8</sub><sup>+</sup> ur<sub>7</sub> R<sub>3,16</sub><sup>+</sup> nr<sub>10</sub> R<sub>12,14</sub><sup>+</sup> ur<sub>13</sub>; The 0-Framed Trefoil

(Do[z2 = z2 // m<sub>1,k</sub> →<sub>1</sub>, {k, 2, 16}];  
 z2 = z2 / . a<sub>-1</sub> ⇒ a)

$$\mathbb{E} \left[ -1 + \frac{1}{t} + t, \theta, \theta, 16 + \frac{2c}{t^4} - \frac{1}{t^3} - \frac{6c}{t^3} + \frac{4}{t^2} + \frac{10c}{t^2} - \frac{10}{t} - \frac{8c}{t} - 18t + 8ct + 14t^2 - 10ct^2 - 7t^3 + 6ct^3 + 2t^4 - 2ct^4 + 2vw - \frac{2vw}{t^4} + \frac{4vw}{t^3} - \frac{6vw}{t^2} + \frac{2vw}{t} - 6tvw + 4t^2vw - 2t^3vw \right]$$



Questions and To Do List. • Clean up and write up. • Implement well, compute for everything in sight. • Why are our quantities polynomials rather than just rational functions? • Bounds on their degrees? • Their integrality (Z) properties? • Can everything be re-stated using integrals (∫)? • Find the 2-variable version (for knots). How complex is it? • What about links / closed components? • Fully digest the “expansion” theorem; include cuaps. • Explore the (non-)dependence on R. • Is there a canonical R? • What does “group like” mean? • Strand removal? Strand doubling? Strand reversal? • Say something about knot genus. • Find the EK/AT/KV “vertex”. • Use as a playground to study associators/braidors. • Restate in topological language. • Study the associated (v-)braid representations. • Study mirror images and the b<sup>+</sup> ↔ b<sup>-</sup> involution. • Study ribbon knots. • Make precise the relationship with Γ-calculus and Alexander. • Relate to the coloured Jones polynomial. • Relate with “ordinary” q-algebra. • k-smidgen sl<sub>n</sub>, etc. • Are there “solvable” CYBE algebras not arising from semi-simple algebras? • Categorify and appease the Gods.

817