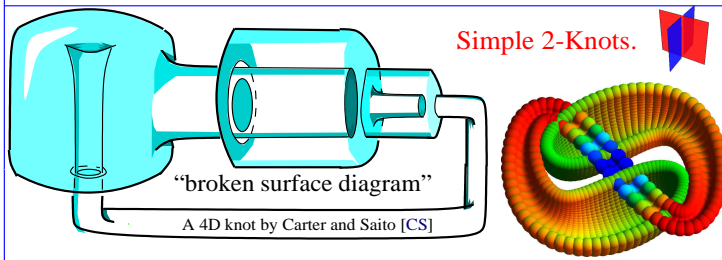
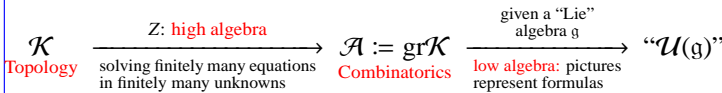


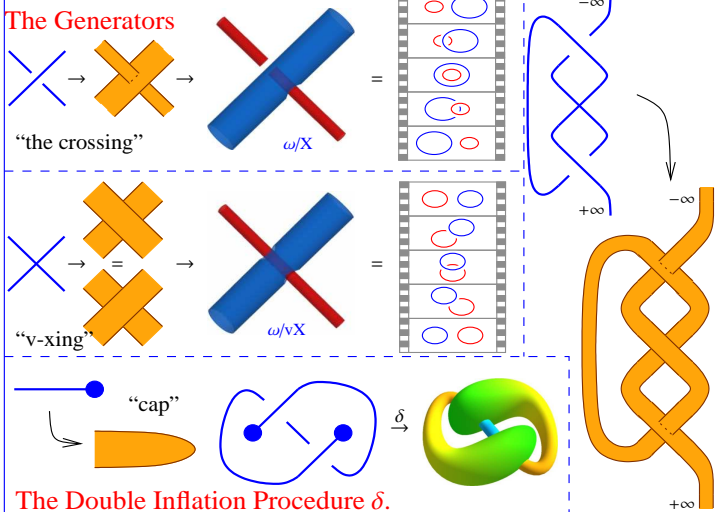


**Abstract.** We will repeat the 3D story of the previous 3 talks one dimension up, in 4D. Surprisingly, there's more room in 4D, and things get easier, at least when we restrict our attention to "w-knots", or to "simply-knotted 2-knots". But even then there are intricacies, and we try to go beyond simply-knotted, we are completely confused.

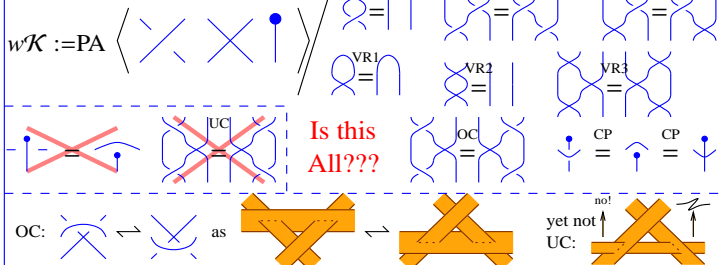
**Recall.**



**The Generators**

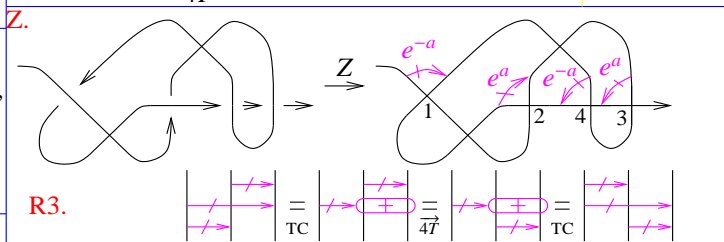
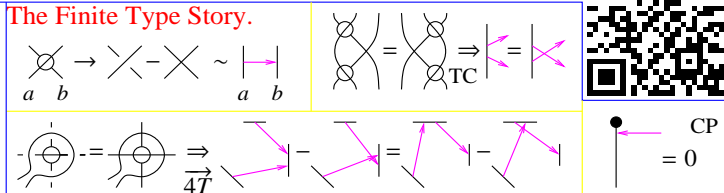


**w-Knots.**

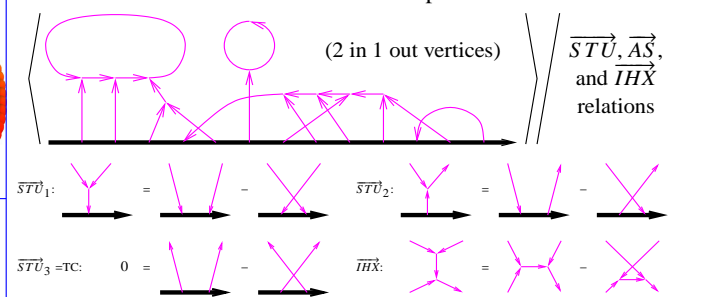


**A Big Open Problem.**  $\delta$  maps w-knots onto simple 2-knots. To what extent is it a bijection? What other relations are required? In other words, **find a simple description of simple 2-knots**. Kawachi [Ka] may already know the answer.

**The Finite Type Story.**



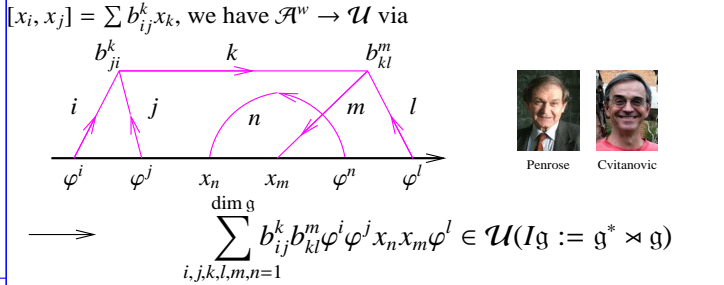
**The Bracket-Rise Theorem.**  $\mathcal{A}^w$  is isomorphic to



**Corollaries.** (1) Only wheels and isolated arrows persist:

$\mathcal{A}^w(\uparrow_n) \cong \mathcal{U}(FL(n)_{lb}^n \ltimes CW(n))$  and  $\zeta := \log Z \in FL(n)^n \times CW(n)$  has completely explicit formulas using natural FL/CW operations [BN].  
 (2) Related to f.d. Lie algebras!

**Low Algebra.** With  $(x_i)$  and  $(\varphi^j)$  dual bases of  $\mathfrak{g}$  and  $\mathfrak{g}^*$  and with



**Differential Ops.** We can also interpret  $\hat{\mathcal{U}}(I\mathfrak{g})$  as tangential differential operators on  $\text{Fun}(\mathfrak{g})$ :

$\varphi \in \mathfrak{g}^*$  becomes a multiplication operator, and  $x \in \mathfrak{g}$  becomes a tangential derivation, in the direction of the action of  $\text{ad } x$ :  $(x\varphi)(y) := \varphi([x, y])$ .

**Too easy so far!** Yet once you add “foam vertices”, it gets related to the Kashiwara-Vergne problem [KV] as told by Alekseev-Torossian [AT]:

