From $V$ to $F$ to KV following [AT].

```
logF = Λ[V][[1]] // dσ[{x, y} → {y, x}];
logF // EulerE // adSeries[(e^ad - 1)/ad, logF, tb]
```

⟨x → LS[y/2, xy/6, 1/24 xyy, −1/180 xxxy + 1/80 xxyy + 1/360 xyyy,
−1/720 xxxyy + 1/240 xxyyy + 1/240 xyxyy + 1/720 xxyxy − xyyyy/1440, xxxxxy/5040 − xxxxyy/1344 + 13 xxxyyy/15120 + 1/840 xxyxyy + xxxyxy/3360 + xxyyyy/6720 + xyxyyy/1260 + xxyyxy/1680 − xyyyyy/10080, …],
y → LS[0, xy/12, 1/24 xyy, −1/360 xxxy + 1/120 xxyy + 1/180 xyyy,
−1/720 xxxyy + 1/240 xxyyy + 1/240 xyxyy + 1/720 xxyxy − xyyyy/1440, xxxxxy/10080 − xxxxyy/2016 + xxxyyy/1890 + xxyxyy/1120 + xxxyxy/5040 + xxyyyy/2520 + 1/840 xyxyyy + xxyyxy/1260 − xyyyyy/5040, …]⟩

---

```
Φs[2, 1] = Φs[3, 1] = Φs[3, 2] = 0;
Φs[3, 1, 2] = 1/24; Φ = DKS[3, Φs];
SeriesSolve[Φ,
  (Φ^σ[3,2,1] ≡ −Φ) ∧
   (Φ ** Φ^σ[1,23,4] ** Φ^σ[2,3,4] ≡ Φ^σ[12,3,4] ** Φ^σ[1,2,34])];
Φ (* Can raise degree to 10 *)
```

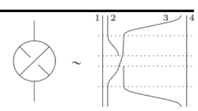
Solving for an associator Φ.

SeriesSolve::ArbitrarilySetting : In degree 3 arbitrarily setting {Φs[3, 1, 1, 2] → 0}.
SeriesSolve::ArbitrarilySetting : In degree 5 arbitrarily setting {Φs[3, 1, 1, 1, 1, 2] → 0}.

DKS[0, 1/24 t13 t23, 0, −7 t13 t23 t23 t23/5760 + 7 t13 t13 t23 t23/5760 − t13 t13 t13 t23/1440,
0, 31 t13 t23 t23 t23 t23 t23/967680 − 157 t13 t13 t23 t23 t13 t23/1935360 − 31 t13 t23 t13 t23 t23 t23/387072 − 31 t13 t13 t23 t23 t23 t23/483840 + 11 t13 t13 t13 t23 t13 t23/290304 + 31 t13 t13 t23 t13 t23 t23/725760 + 83 t13 t13 t13 t23 t23 t23/967680 − 13 t13 t13 t13 t13 t23 t23/241920 + t13 t13 t13 t13 t13 t23/60480, …]

---

The “buckle” $Z_B$, from Φ.



```
R = DKS[t[1, 2]/2];
Z_B = (−Φ)^σ[13,2,4] ** Φ^σ[1,3,2] ** R^σ[2,3] ** (−Φ)^σ[1,2,3] **
   Φ^σ[12,3,4];
Z_B@{4}
```

DKS[t23/2, −1/12 t13 t23 − 1/24 t14 t24 + 1/24 t14 t34 + 1/12 t24 t34,
0, t13 t23 t23 t23/5760 + 7 t14 t24 t24 t24/5760 + t14 t34 t24 t24/1920 − t14 t34 t34 t24/1920 − 7 t14 t34 t34 t34/5760 − t24 t34 t34 t34/5760 + t14 t24 t34 t24/1920 + t14 t24 t14 t34/1920 − t14 t34 t24 t34/1920 − 1/720 t13 t13 t23 t23 + 1/720 t13 t13 t13 t23 − 7 t14 t14 t24 t24/5760 + 7 t14 t14 t34 t34/5760 − t14 t24 t34 t34/5760 + t14 t14 t14 t24/1440 − t14 t14 t14 t34/1440 − 1/960 t14 t14 t24 t34 + t14 t24 t24 t34/5760 − 1/960 t24 t24 t34 t34 − t24 t24 t24 t34/5760, …]

---

$V$ from $Z_B$, following [AET, BND].

```
(El[Z_B // αMap[1, 2, 3, 4], CWS[0]] // Γ // tη^1 // tη^3 //
   hη^2 // hη^4 // hσ[{3} → {2}] // tσ[{2, 4} → {1, 2}])[[
 1]]
```

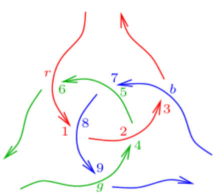
⟨1 → LS[0, −12/24, 0, 71112/5760 − 71122/5760 + 1222/1440, 0,
−31 111112/967680 + 31 111122/483840 − 83 111222/967680 − 31 112122/725760 − 31 112112/645120 + 13 112222/241920 + 101 121222/1451520 + 527 112212/5806080 − 122222/60480, …],
2 → LS[1/2, −12/12, 0, 1112/5760 − 1/720 1122 + 1/720 1222,
−11112/7680 + 11122/3840 − 11212/6912,
−111112/645120 + 23 111122/483840 − 13 111222/161280 − 112122/22680 − 41 112112/580608 + 112222/15120 + 121222/12096 + 71 112212/483840 − 122222/30240, …]⟩

---

The Borromean tangle.

```
Rs[a_, b_] := Es[⟨a → LS[0], b → LS[LW@a]⟩, CWS[0]];
iRs[a_, b_] := Es[⟨a → LS[0], b → −LS[LW@a]⟩, CWS[0]];
ξ = iRs[r, 6] Rs[2, 4] iRs[g, 9] Rs[5, 7] iRs[b, 3] Rs[8, 1];
```

```
Do[ξ = ξ // dm[r, k, r], {k, 1, 3}];
Do[ξ = ξ // dm[g, k, g], {k, 4, 6}];
Do[ξ = ξ // dm[b, k, b], {k, 7, 9}];
{ξ[[1]]_r@{5}, ξ[[2]]@{5}} // Print
```





;

{LS[0, bg, 1/2 bbg + bgr + 1/2 bgg,
1/6 bbbg + 1/2 bbgr + 1/2 bggr + 1/4 bbgg + 1/2 bgrr + 1/6 bggg,
1/24 bbbbg + 1/6 bbbgr + 1/4 bbggr + 1/12 bbbgg +
1/4 bbgrr + 1/6 bgggr + 1/4 bggrr − bbgrg +
1/12 bbggg − 2 bbrgg + 1/6 bgrrr + 1/2 bgbgr −
bgbrg − 1/12 bbgbg − 1/2 bgrgr + 1/24 bgggg, …],
CWS[0, 0, 2 bgr, bbgr − bgbr + bggr − bgrg + bgrr − brgr, bbbgr/3 −
bbgbr/2 + bbggr/2 + bbgrg/2 + bbgrr/2 + bbrbg/2 − 3 bbrgr/2 + bgbrr/2 − 3 bggbr/2 + bgggr/3 −
bggrg/2 + bggrr/2 + bgrgg/2 − 3 bgrrg/2 + bgrrr/3 + brggr/2 − brgrr/2 + brrgr/2, …]}

---

References.


[AT] A. Alekseev and C. Torossian, *The Kashiwara-Vergne conjecture and Drinfeld's associators,* Annals of Mathematics **175** (2012) 415–463, arXiv:0802.4300.

[AET] A. Alekseev, B. Enriquez, and C. Torossian, *Drinfeld's associators, braid groups and an explicit solution of the Kashiwara-Vergne equations,* Publications Mathématiques de L'IHÉS, **112-1** (2010) 143–189, arXiv:0903.4067

[BND] D. Bar-Natan and Z. Dancso, *Finite Type Invariants of W-Knotted Objects I-IV,* ωεβ/WKO1, ωεβ/WKO2, ωεβ/WKO3, ωεβ/WKO4, and arXiv:1405.1956, arXiv:1405.1955, arXiv: not.yet×2.


Warning. Fidgety!

Video and more at `http://www.math.toronto.edu/~drorbn/Talks/LesDiablerets-1508/`