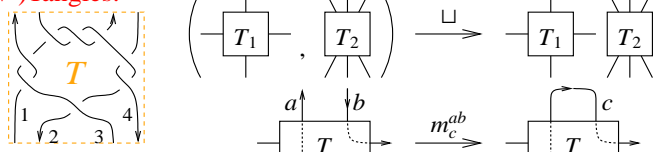




Abstract. The value of things is inversely correlated with their computational complexity. "Real time" machines, such as our brains, only run linear time algorithms, and there's still a lot we don't know. Anything we learn about things doable in linear time is truly valuable. Polynomial time we can in-practice run, even if we have to wait; these things are still valuable. Exponential time we can play with, but just a little, and exponential things must be beautiful or philosophically compelling to deserve attention. Values further diminish and the aesthetic-or-philosophical bar further rises as we go further slower, or un-computable, or ZFC-style intrinsically infinite, or large-cardinalish, or beyond.

I will explain some things I know about polynomial time knot polynomials and explain where there's more, within reach.

(v-)Tangles.



Why Tangles?

- Finitely presented. (meta-associativity: $m_a^{ab} // m_a^{ca} = m_b^{bc} // m_a^{ab}$)
 - Divide and conquer proofs and computations.
 - "Algebraic Knot Theory": If K is ribbon, $z(K) \in \{cl_2(\zeta) : cl_1(\zeta) = 1\}$. $U \in \mathcal{T}_n$
 - "Genus and crossing number are also definable properties). cl_1 : trivial cl_2 : ribbon $K \in \mathcal{T}_1$
- Faster is better, leaner is meaner!

Theorem 1. $\exists!$ an invariant z_0 : {pure framed S -component tangles} $\rightarrow \Gamma_0(S) := R \times M_{S \times S}(R)$, where $R = R_S = \mathbb{Z}((T_a)_{a \in S})$ is the ring of rational functions in S variables, intertwining

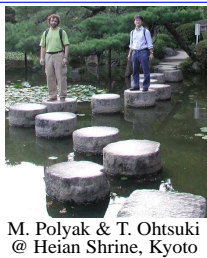
$$\begin{pmatrix} \omega_1 & S_1 \\ S_1 & A_1 \end{pmatrix}, \begin{pmatrix} \omega_2 & S_2 \\ S_2 & A_2 \end{pmatrix} \xrightarrow{\sqcup} \begin{pmatrix} \omega_1 \omega_2 & S_1 & S_2 \\ S_1 & A_1 & 0 \\ S_2 & 0 & A_2 \end{pmatrix}$$

$$\begin{matrix} \omega & a & b & S \\ a & \alpha & \beta & \theta \\ b & \gamma & \delta & \epsilon \\ S & \phi & \psi & \Xi \end{matrix} \xrightarrow[m_c^{ab}]{T_a, T_b \rightarrow T_c} \begin{matrix} \mu\omega & c & S \\ c & \gamma + \alpha\delta/\mu & \epsilon + \delta\theta/\mu \\ S & \phi + \alpha\psi/\mu & \Xi + \psi\theta/\mu \end{matrix}$$

and satisfying $(|a; a \nearrow b, b \nearrow a) \xrightarrow{z_0} \begin{pmatrix} 1 & a \\ a & 1 \end{pmatrix}; \begin{pmatrix} 1 & a & b \\ a & 1 & 1 - T_a^{\pm 1} \\ b & 0 & T_a^{\pm 1} \end{pmatrix}$

In Addition • The matrix part is just a stitching formula for Burau/Gassner [LD, KLV, CT].

- $K \mapsto \omega$ is Alexander, mod units.
- $L \mapsto (\omega, A) \mapsto \omega \det(A - I)/(1 - T')$ is the MVA, mod units.
- The fastest Alexander algorithm I know.
- There are also formulas for strand deletion, reversal, and doubling.
- Every step along the computation is the invariant of something.
- Extends to and more naturally defined on v/w-tangles.
- Fits in one column, including propaganda & implementation.



Implementation key idea:

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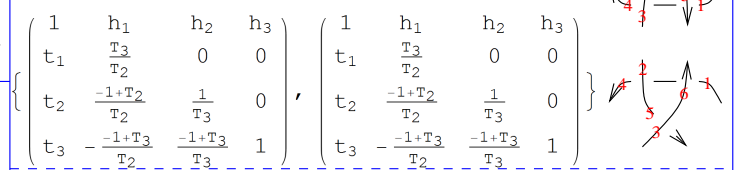
F := F[\omega_1, \lambda_1] F[\omega_2, \lambda_2] := F[\omega_1 * \omega_2, \lambda_1 * \lambda_2];
m_{a,b} \rightarrow c := Module[\alpha, \beta, \gamma, \delta, \epsilon, \phi, \psi, \Xi, \mu];
\begin{matrix} \alpha & \beta & \theta \\ \gamma & \delta & \epsilon \\ \phi & \psi & \Xi \end{matrix} = \begin{matrix} \partial_{\alpha, \beta, \gamma} \lambda & \partial_{\alpha, \beta, \gamma} \lambda & \partial_{\alpha, \beta, \gamma} \lambda \\ \partial_{\gamma, \delta, \epsilon} \lambda & \partial_{\gamma, \delta, \epsilon} \lambda & \partial_{\gamma, \delta, \epsilon} \lambda \\ \partial_{\phi, \psi, \Xi} \lambda & \partial_{\phi, \psi, \Xi} \lambda & \lambda \end{matrix} / (t|h)_{a,b} \rightarrow 0;
\Gamma[\mu = 1 - \beta] \omega, \{t_c, 1\} \cdot (\gamma + \alpha\delta/\mu \epsilon + \delta\theta/\mu) \cdot (h_c, 1) / (\phi + \alpha\psi/\mu \Xi + \psi\theta/\mu) // RCollect];
M = Outer[Factor[\partial_{h_c, t_c} \lambda], \{h, t_c\}, a, \omega];
M = Prepend[M, t_c & / \theta S] // Transpose;
M = Prepend[M, Prepend[h_c & / \theta S, \omega]];
M // MatrixForm];
  
```

Meta-Associativity

$$\xi = \Gamma[\omega, \{t_1, t_2, t_3, t_s\} \cdot \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \theta_1 \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \theta_2 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & \theta_3 \\ \phi_1 & \phi_2 & \phi_3 & \Xi \end{pmatrix} \cdot \{h_1, h_2, h_3, h_s\}];$$

$$(\xi // m_{12 \rightarrow 1} // m_{13 \rightarrow 1}) = (\xi // m_{23 \rightarrow 2} // m_{12 \rightarrow 1})$$

True $\xrightarrow{R3}$... divide and conquer!
 $\{Rm_{51} Rm_{62} Rp_{34} // m_{14 \rightarrow 1} // m_{25 \rightarrow 2} // m_{36 \rightarrow 3}, Rp_{61} Rm_{24} Rm_{35} // m_{14 \rightarrow 1} // m_{25 \rightarrow 2} // m_{36 \rightarrow 3}\}$

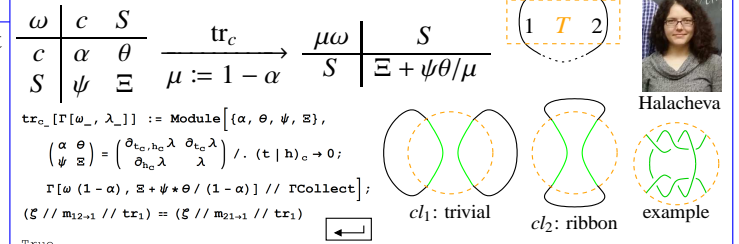


$$z = Rm_{12,1} Rm_{27} Rm_{83} Rm_{4,11} Rp_{16,5} Rp_{6,13} Rp_{14,9} Rp_{10,15};$$

$$Do[z = z // m_{1k \rightarrow 1}, \{k, 2, 16\}];$$

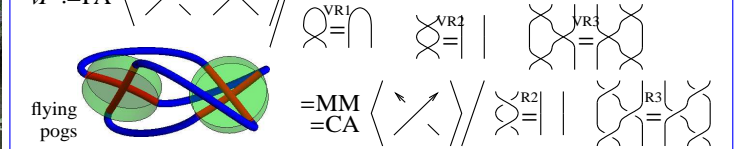
$$z = (11 - \frac{1}{T_1^3} + \frac{4}{T_1^2} - \frac{8}{T_1} - 8T_1 + 4T_1^2 - T_1^3) h_1$$

Closed Components. The Halacheva trace tr_c satisfies $m_c^{ab} // tr_c = m_c^{ba} // tr_c$ and computes the MVA for all links in the atlas, but its domain is not understood:



Weaknesses. • m_c^{ab} and tr_c are non-linear. • The product ωA is always Laurent, but my current proof takes induction with exponentially many conditions. • I still don't understand tr_c , "unitarity", the algebra for ribbon knots. **Where does it come from?**

v-Tangles.



Let $\mathcal{I} := \langle \times, - \times \rangle$. Then $\mathcal{A}^v := \prod I^n / I^{n+1} = \text{"universal } \mathcal{U}(Dg)^{\otimes S} = \dots$
 (Also IHX)
 Fine print: No sources no sinks, AS vertices, internally acyclic, deg = (#vertices)/2.

Likely Theorem. [EK, En] There exists a homomorphic expansion (universal finite type invariant) $Z: v\mathcal{T} \rightarrow \mathcal{A}^v$. (issues suppressed)
Too hard! Let's look for "meta-monoid" quotients.

