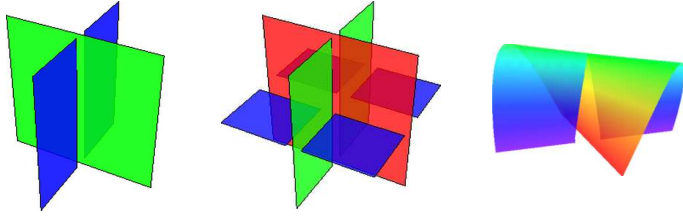
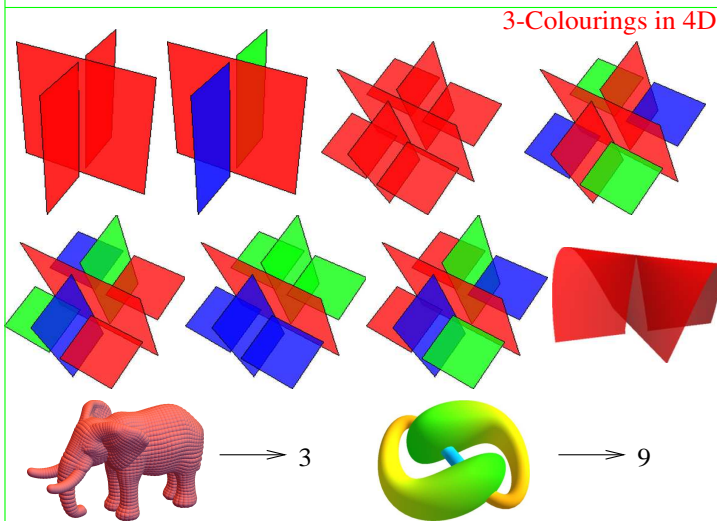
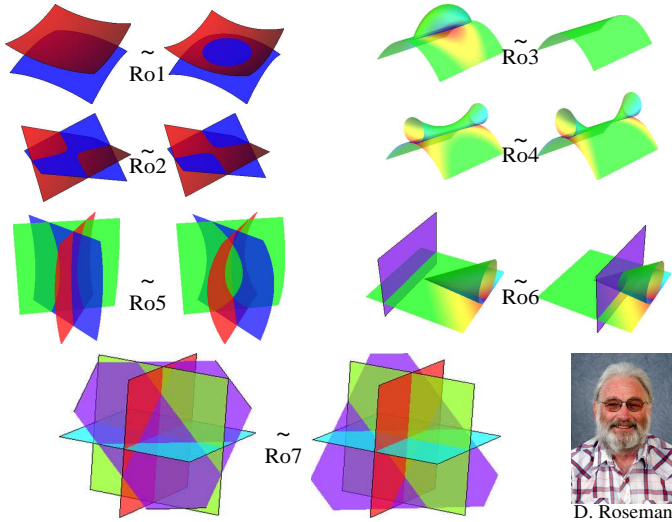


**Theorem.** Every 2-knot can be represented by a “broken surface diagram” made of the following basic ingredients,



... and any two representations of the same knot differ by a sequence of the following “Roseman moves”:

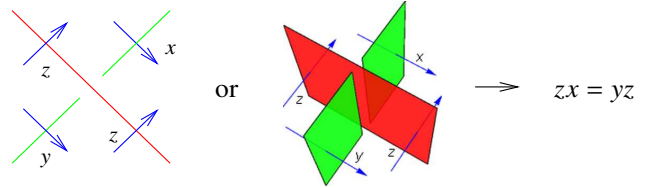


**A Knot Table**

There are many more!

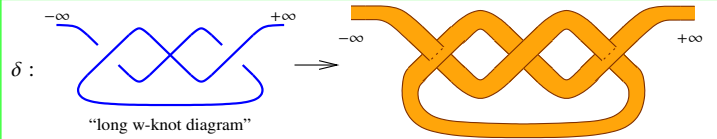

$\omega/KT$

**A Stronger Invariant.** There is an assignment of groups to knots / 2-knots as follows. Put an arrow “under” every un-broken curve / surface in a broken curve / surface diagram and label it with the name of a group generator. Then mod out by relations as below.

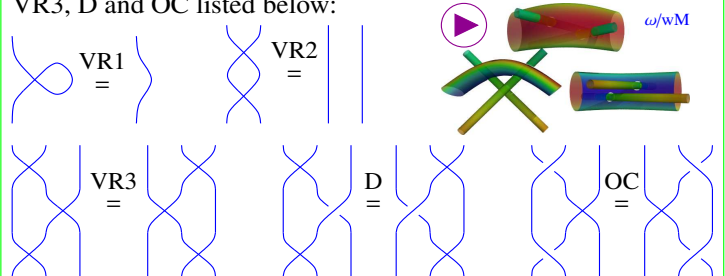


**Facts.** The resulting “Fundamental group”  $\pi_1(K)$  of a knot / 2-knot  $K$  is a very strong but not very computable invariant of  $K$ . Though it has computable projections; e.g., for any finite  $G$ , count the homomorphisms from  $\pi_1(K)$  to  $G$ .

**Exercise.** Show that  $|\text{Hom}(\pi_1(K) \rightarrow S_3)| = \lambda(K) + 3$ .



**Satoh's Conjecture.** (Satoh, *Virtual Knot Presentations of Ribbon Torus-Knots*, *J. Knot Theory and its Ramifications* **9** (2000) 531–542). Two long w-knot diagrams represent via the map  $\delta$  the same simple long 2D knotted tube in 4D iff they differ by a sequence of R-moves as above and the “w-moves” VR1–VR3, D and OC listed below:



**Some knot theory books.**

- Colin C. Adams, *The Knot Book, an Elementary Introduction to the Mathematical Theory of Knots*, American Mathematical Society, 2004.
- Meike Akveld and Andrew Jobbings, *Knots Unravalled, from Strings to Mathematics*, Arbelos 2011.
- J. Scott Carter and Masahico Saito, *Knotted Surfaces and Their Diagrams*, American Mathematical Society, 1997.
- Peter Cromwell, *Knots and Links*, Cambridge University Press, 2004.
- W.B. Raymond Lickorish, *An Introduction to Knot Theory*, Springer 1997.

