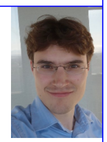
Work in Progress!

The Brute and the Hidden Paradise



van der Veen

Local Algebra (with van der Veen) Much can be re-formulated as (non-standard) "quantum algebra" for the 4D Lie algebra $\mathfrak{g} = \langle b, c, u, w\rangle$ over $\mathbb{Q}[\epsilon]/(\epsilon^2 = 0)$, with b central and $[w, c] = w$, $[c, u] = u$, and $[u, w] = b - 2\epsilon c$. The key: $a_{ij} = (b_i - \epsilon c_i)c_j + u_i w_j$ in $\mathcal{U}(\mathfrak{g})^{\otimes\{i,j\}}$.

Some (new) representationss of the (v-)braid groups. ωεβ/Reps

Burau (old)

```
B_{i_,j_}[ℰ_] := ℰ /. v_j :→ (1 - t) v_i + t v_j
```

…testing R3

```
Column@{lhs = {v1, v2, v3} // B1,2 // B1,3 // B2,3,
  rhs = {v1, v2, v3} // B2,3 // B1,3 // B1,2,
  lhs - rhs // Expand}

{v1, (1 - t) v1 + t v2, (1 - t) v1 + t ((1 - t) v2 + t v3)}
{v1, (1 - t) v1 + t v2,
 (1 - t) ((1 - t) v1 + t v2) + t ((1 - t) v1 + t v3)}
{0, 0, 0}
```

Gassner (old)

```
G_{i_,j_}[ℰ_] := ℰ /. v_j :→ (1 - t_i) v_i + t_i v_j
```

…Overcrossings Commute (OC):

```
Column@{lhs = {v1, v2, v3} // G1,2 // G1,3,
  Expand[lhs - ({v1, v2, v3} // G1,3 // G1,2)]}

{v1, (1 - t1) v1 + t1 v2, (1 - t1) v1 + t1 v3}
{0, 0, 0}
```

…Undercrossings Commute (UC):

```
Column@{lhs = {v1, v2, v3} // G1,3 // G2,3,
  rhs = {v1, v2, v3} // G2,3 // G1,3,
  lhs - rhs // Expand}

{v1, v2, (1 - t1) v1 + t1 ((1 - t2) v2 + t2 v3)}
{v1, v2, (1 - t2) v2 + t2 ((1 - t1) v1 + t1 v3)}
{0, 0, v1 - t1 v1 - t2 v1 + t1 t2 v1 - v2 + t1 v2 + t2 v2 - t1 t2 v2}
```

Gassner Plus (new?)

```
GP_{i_,j_}[ℰ_] := Expand[ℰ /. {u_j :→ (1 - t_i) u_i + t_i u_j,
    f_. v_j :→ f (1 - t_i) v_i + f t_i v_j + (t_i - 1) (t_i ∂_{t_i} f - t_j ∂_{t_j} f) u_i +
      f t_i u_i }];
bas = {f[t1, t2, t3] v1, f[t1, t2, t3] v2, f[t1, t2, t3] v3,
   u1, u2, u3};
```

…R3 (left)

```
Short[lhs = bas // GP1,2 // GP1,3 // GP2,3, 2]

{f[t1, t2, t3] v1, f[t1, t2, t3] t1 u1 + f[t1, t2, t3] v1 -
  f[t1, t2, t3] t1 v1 + <<6>> + t1^2 u1 f^(1,0,0)[t1, t2, t3],
 <<1>> + <<19>> + <<1>>, <<1>>, u1 - t1 u1 + t1 u2,
 u1 - t1 u1 + t1 u2 - t1 t2 u2 + t1 t2 u3}
```

…R3 (rest)

```
(bas // GP2,3 // GP1,3 // GP1,2) - lhs
{0, 0, 0, 0, 0, 0}
```

…OC

```
(bas // GP1,2 // GP1,3) - (bas // GP1,3 // GP1,2)
{0, 0, 0, 0, 0, 0}
```

Question. Does Gassner Plus factor through Gassner?

Turbo-Gassner (new!)

```
Kδ_{i_,j_} := KroneckerDelta[i, j];
TG_{i_,j_}[ℰ_] := Expand[ℰ /. {
    f_. v_k_ :→ Plus[f v_k /. v_j → (1 - t_i) v_i + t_i v_j,
       (1 - t_i^-1) (t_i ∂_{t_i} f - t_j ∂_{t_j} f) *
        (u_k /. u_j → (1 - t_i) u_i + t_i u_j) * u_i w_j,
       Kδ_{k,i} f (u_j - u_i) u_i w_j],
    u_j → (1 - t_i) u_i + t_i u_j,
    w_i → w_i + (1 - t_i^-1) w_j, w_j → t_i^-1 w_j}];
bas = {f[t1, t2, t3] v1, f[t1, t2, t3] v2, f[t1, t2, t3] v3,
   u1, u2, u3, w1, w2, w3};
```

Satisfies R3…

…OC

```
(bas // TG1,2 // TG1,3) - (bas // TG1,3 // TG1,2)

{0, -f[t1, t2, t3] u1 u2 w3 + f[t1, t2, t3] t1 u1 u2 w3 +
  f[t1, t2, t3] u1 u3 w3 - f[t1, t2, t3] t1 u1 u3 w3,
 -f[t1, t2, t3] u1 u2 w2 + f[t1, t2, t3] t1 u1 u2 w2 +
  f[t1, t2, t3] u1 u3 w2 -
  f[t1, t2, t3] t1 u1 u3 w2, 0, 0, 0, 0, 0, 0}
```

Turbo-Burau (new!)

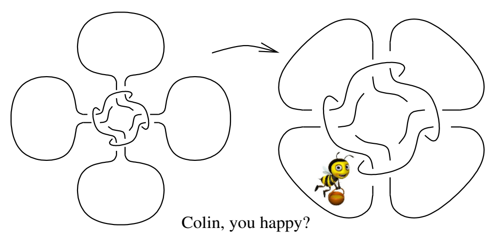
```
η /: η[i_]^2 = 0; η /: η[i_] η[j_] = 0;
TB_{i_,j_}[ℰ_] :=
  Expand[ℰ /. {
     f_. v_k_ :→ Plus[f v_k /. v_j → (1 - t - η[i]) v_i + (t + η[i]) v_j,
       (t - 1) (Coefficient[f, η[i]] - Coefficient[f, η[j]]) *
        (u_k /. u_j → (1 - t) u_i + t u_j) * u_i w_j,
       Kδ_{k,i} (f /. _η → 0) (u_j - u_i) u_i w_j],
     u_j → (1 - t) u_i + t u_j,
     w_i → w_i + (1 - t^-1) w_j, w_j → t^-1 w_j}];
ff = f0 + f1 η[1] + f2 η[2] + f3 η[3];
bas = {ff v1, ff v2, ff v3, u1^2 w1, u1^2 w2, u1, u2, u3, w1, w2, w3};
```

…OC

```
(bas // TB1,2 // TB1,3) - (bas // TB1,3 // TB1,2)

{0, -f0 u1 u2 w3 + t f0 u1 u2 w3 + f0 u1 u3 w3 - t f0 u1 u3 w3,
 -f0 u1 u2 w2 + t f0 u1 u2 w2 + f0 u1 u3 w2 - t f0 u1 u3 w2,
 0, 0, 0, 0, 0, 0, 0, 0}
```

Flower Surgery Theorem. A knot is ribbon iff it is the result of n-petal flower surgery (from thin petals to wide petals) on an n-componenet unlink, for some n.



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"God created the knots, all else in topology is the work of mortals."

Leopold Kronecker (modified)

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Video and more at http://www.math.toronto.edu/~drorbn/Talks/Greece-1607/