

Conversions  $\mathcal{A} \leftrightarrow \Gamma$ :

```

Γ@A[is_, os_, cs_, w_] := Module[{i, j, ω = Coefficient[w, Wedge[.]],
  Γ[is, os, cs, ω, Sum[Cancel[-Coefficient[w, Xj ^ ξ1] ξ1 Xj / ω],
    {i, is}, {j, os}]]];
A@Γ[is_, os_, cs_, ω_, λ_] :=
  A[is, os, cs, Expand[ω WExp[Expand[λ] /. ξa_ Xb_ -> ξa ^ Xb]]];

```

The conversions are inverses of each other:

```

Γ[{1, 2, 3}, {1, 2, 3}, {X1 -> τ1, X2 -> τ2, X3 -> τ3, ξ1 -> τ1, ξ2 -> τ2, ξ3 -> τ3},
  ω, a11 X1 ξ1 + a12 X2 ξ1 + a13 X3 ξ1 + a21 X1 ξ2 + a22 X2 ξ2 + a23 X3 ξ2 + a31 X1 ξ3 +
  a32 X2 ξ3 + a33 X3 ξ3];

```

```

Γ@A@Γ = γ

```

True

The conversions commute with contractions:

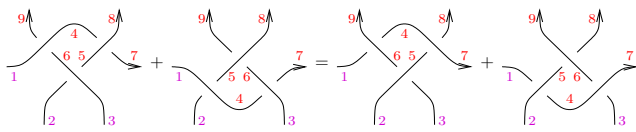
```

Γ@c3,3@A@Γ = c3,3@γ

```

True

### Conway's Third Identity



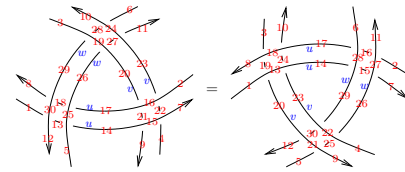
Sorry,  $\Gamma$  has nothing to say about that...

### References

- J. Archibald, *The Multivariable Alexander Polynomial on Tangles*, University of Toronto Ph.D. thesis, 2010, <http://drorbn.net/mo21/AT>.
- J. H. Conway, *An Enumeration of Knots and Links, and some of their Algebraic Properties*, Computational Problems in Abstract Algebra (Proc. Conf., Oxford, 1967), Pergamon, Oxford, 1970, 329–358.
- Z. Dancso, I. Halacheva, and M. Robertson, *Circuit Algebras are Wheeled Props*, J. Pure and Appl. Alg., to appear, [arXiv:2009.09738](https://arxiv.org/abs/2009.09738).
- I. Halacheva, *Alexander Type Invariants of Tangles, Skew Howe Duality for Crystals and The Cactus Group*, University of Toronto Ph.D. thesis, 2016, <http://drorbn.net/mo21/HT>.
- I. Halacheva, *Alexander Type Invariants of Tangles*, [arXiv:1611.09280](https://arxiv.org/abs/1611.09280).

Thank You!

### The Naik-Stanford Double Delta Move (again)



```

Timing[Γ@{X6,10,28,24[w,v], X28,3,29,19[w,v], X26,20,27,19[w,v], X27,23,11,24[w,v],
  X1,12,13,30[u,w], X13,5,14,25[u,w], X17,26,18,25[u,w], X18,29,8,30[u,w],
  X4,7,22,15[v,u], X22,2,23,16[v,u], X20,17,21,16[v,u], X21,14,9,15[v,u]} ≡
  Γ@{X5,9,25,21[w,v], X25,4,26,22[w,v], X29,23,30,22[w,v], X30,20,12,21[w,v],
  X2,11,16,27[u,w], X16,6,17,28[u,w], X14,29,15,28[u,w], X15,26,7,27[u,w],
  X3,8,19,18[v,u], X19,1,20,13[v,u], X23,14,24,13[v,u], X24,17,10,18[v,u]}]

```

### What I still don't understand.

- What becomes of  $c_{x,\xi} e^\lambda$  if we have to divide by 0 in order to write it again as an exponentiated quadratic? Does it still live within a very small subset of  $\Lambda(\mathcal{X} \sqcup X)$ ?
- How do cablings and strand reversals fit within  $\mathcal{A}$ ?
- Are there "classicality conditions" satisfied by the invariants of classical tangles (as opposed to virtual ones)?

- M. Markl, S. Merkulov, and S. Shadrin, *Wheeled PROPs, Graph Complexes and the Master Equation*, J. Pure and Appl. Alg. **213-4** (2009) 496–535, [arXiv:math/0610683](https://arxiv.org/abs/math/0610683).
- J. Murakami, *A State Model for the Multivariable Alexander Polynomial*, Pacific J. Math. **157-1** (1993) 109–135.
- S. Naik and T. Stanford, *A Move on Diagrams that Generates S-Equivalence of Knots*, J. Knot Theory Ramifications **12-5** (2003) 717–724, [arXiv:math/9911005](https://arxiv.org/abs/math/9911005).
- Wolfram Language & System Documentation Center, <https://reference.wolfram.com/language/>.