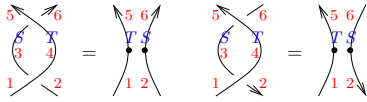


Reidemeister 2



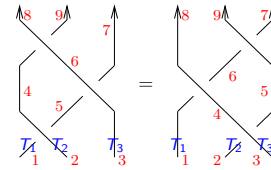
$$\mathcal{A}@\{X_{2,4,3,1}[S, T], \bar{X}_{3,4,6,5}\} \equiv \mathcal{A}@\{P_{1,5}[T], P_{2,6}[S]\}$$

True

$$\mathcal{A}@\{\bar{X}_{3,1,2,4}[S, T], X_{6,5,3,4}\} \equiv \mathcal{A}@\{P_{1,5}[T], P_{6,2}[S]\}$$

True

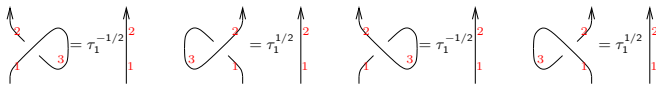
Reidemeister 3



$$\mathcal{A}@\{X_{2,5,4,1}[T_2, T_1], X_{3,7,6,5}[T_3, T_1], X_{6,9,8,4}\} \equiv \mathcal{A}@\{X_{3,5,4,2}[T_3, T_2], X_{4,6,8,1}[T_3, T_1], X_{5,7,9,6}\}$$

True

Reidemeister 1

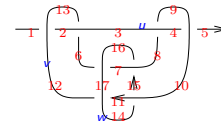


$$\{\mathcal{A}@\{X_{3,3,2,1}\} \equiv \tau_1^{-1/2} \mathcal{A}@\{P_{1,2}\}, \mathcal{A}@\{X_{1,2,3,3}\} \equiv \tau_1^{1/2} \mathcal{A}@\{P_{1,2}\}, \mathcal{A}@\{\bar{X}_{1,3,3,2}\} \equiv \tau_1^{-1/2} \mathcal{A}@\{P_{1,2}\}, \mathcal{A}@\{\bar{X}_{3,1,2,3}\} \equiv \tau_1^{1/2} \mathcal{A}@\{P_{1,2}\}\}$$

{True, True, True, True}

(So we have an invariant, up to rotation numbers).

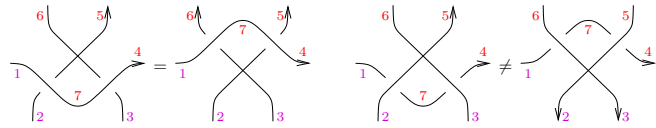
The Relation with the Multivariable Alexander Polynomial



$$\begin{aligned} \text{MVA} &= u^{-1/2} v^{-1/2} w^{-1/2} (u-1)(v-1)(w-1); \\ \text{A} &= \{\bar{X}_{1,12,2,13}[u, v], \bar{X}_{13,2,6,3}, X_{8,4,9,3}, X_{4,10,5,9}, X_{6,17,7,16}[v, w], \\ &\quad X_{15,8,16,7}, \bar{X}_{14,10,15,11}, \bar{X}_{11,17,12,14}\} // \mathcal{A} // \text{Last} // \text{Factor} \\ &\quad \frac{(-1+u)^2(-1+v)(-1+w)(\text{Wedge}[\] - x_5 \wedge \xi_1)}{u v} \\ \text{A} &= u^{-1/2} (u-1) u^0 v^{-1/2} w^{1/2} \text{MVA} (\text{Wedge}[\] - x_5 \wedge \xi_1) \end{aligned}$$

True

Overcrossings Commute but Undercrossings don't



$$\mathcal{A}@\{X_{2,7,5,1}, X_{3,4,6,7}\} \equiv \mathcal{A}@\{X_{3,7,6,1}, X_{2,4,5,7}\}$$

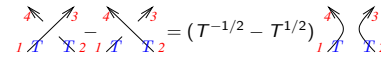
True

$$\mathcal{A}@\{\bar{X}_{1,2,7,5}, \bar{X}_{7,3,4,6}\} \equiv \mathcal{A}@\{\bar{X}_{1,3,7,6}, \bar{X}_{7,2,4,5}\}$$

False

The Conway Relation

(see [Co])



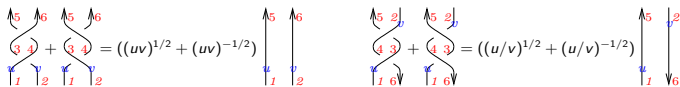
$$\mathcal{A}@\{X_{2,3,4,1}[T, T]\} - \mathcal{A}@\{\bar{X}_{1,2,3,4}[T, T]\} \equiv (T^{-1/2} - T^{1/2}) \mathcal{A}@\{P_{1,4}[T], P_{2,3}[T]\}$$

True



Conway's Second Set of Identities

(see [Co])



$$\mathcal{A}@\{X_{2,4,3,1}[v, u], X_{4,6,5,3}\} + \mathcal{A}@\{\bar{X}_{1,2,4,3}[u, v], \bar{X}_{3,4,6,5}\} \equiv (u^{1/2} v^{1/2} + u^{-1/2} v^{-1/2}) \mathcal{A}@\{P_{1,5}[u], P_{2,6}[v]\}$$

True

$$\mathcal{A}@\{\bar{X}_{4,1,6,3}[v, u], \bar{X}_{3,2,5,4}\} + \mathcal{A}@\{X_{1,6,3,4}[u, v], X_{2,5,4,3}\} \equiv (u^{1/2} v^{-1/2} + u^{-1/2} v^{1/2}) \mathcal{A}@\{P_{1,5}[u], P_{2,6}[v]\}$$

True

Virtual versions (Archibald, [Ar])



$$\mathcal{A}@\{X_{2,3,4,1}\} + \mathcal{A}@\{\bar{X}_{2,1,4,3}\} \equiv (\tau_1^{1/2} + \tau_1^{-1/2}) \mathcal{A}@\{P_{1,3}, P_{2,4}\}$$

True

$$\mathcal{A}@\{\bar{X}_{1,2,3,4}\} + \mathcal{A}@\{X_{1,4,3,2}\} \equiv (\tau_2^{1/2} + \tau_2^{-1/2}) \mathcal{A}@\{P_{1,3}, P_{2,4}\}$$

True