

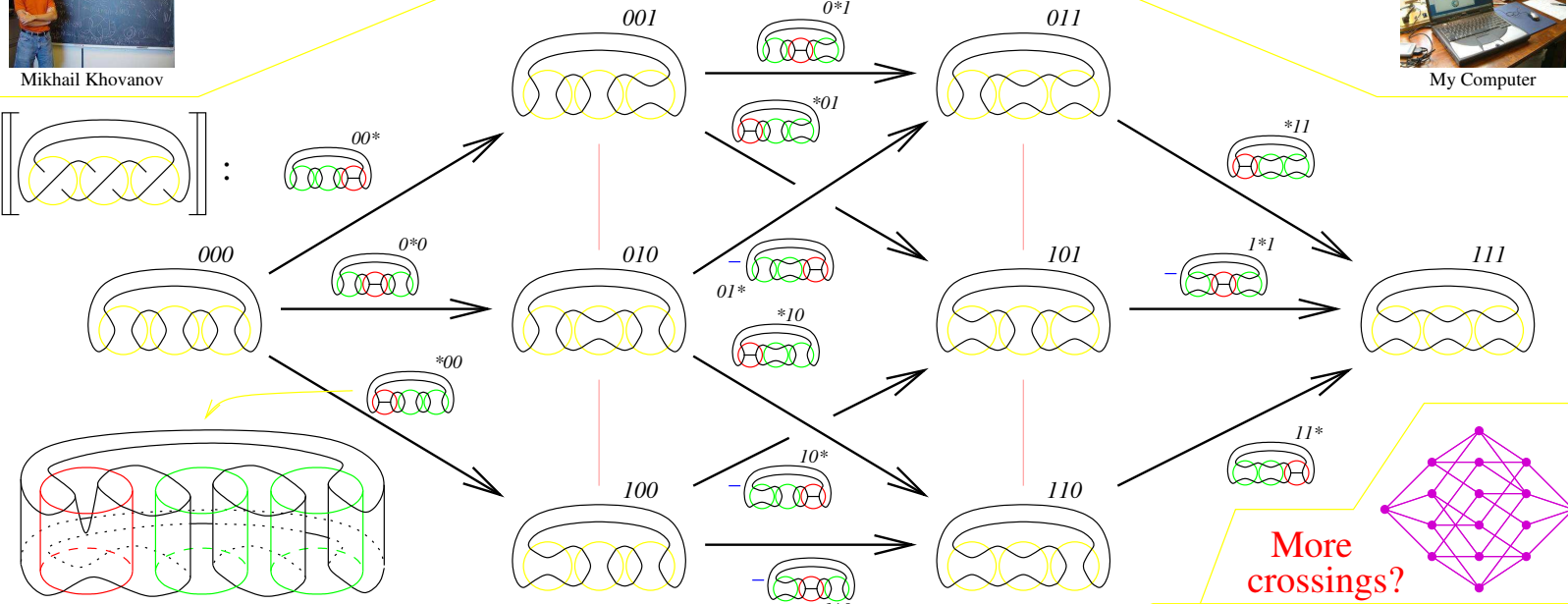
Fast Khovanov Homology Computations



Mikhail Khovanov



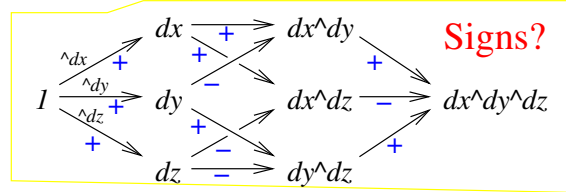
My Computer



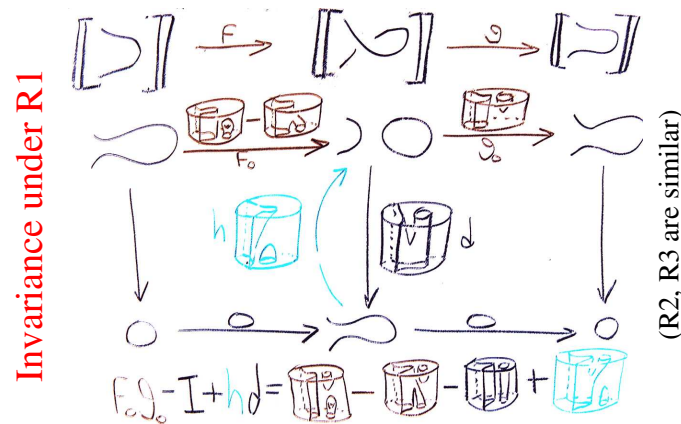
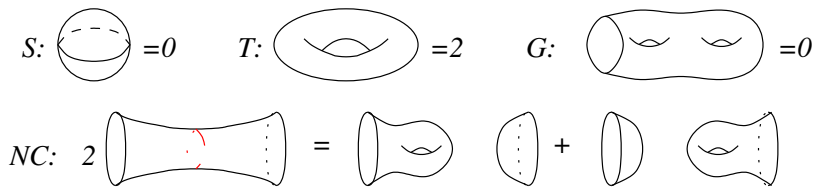
What is it? A cube for each knot/link projection;

Vertices: All fillings of with or with .

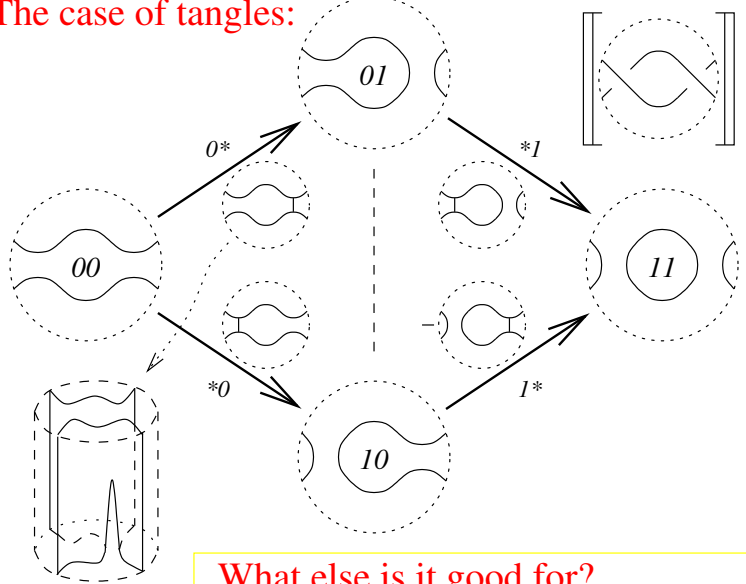
Edges: All fillings of $I \times$ = with $I \times$ = or with $I \times$ = and precisely one .



Where does it live? In $Kom(Mat(\langle Cob \rangle / \{S, T, G, NC\})) / homotopy$
 Kom: Complexes Mat: Matrices Cob: Cobordisms $\langle \dots \rangle$: Formal lin. comb.



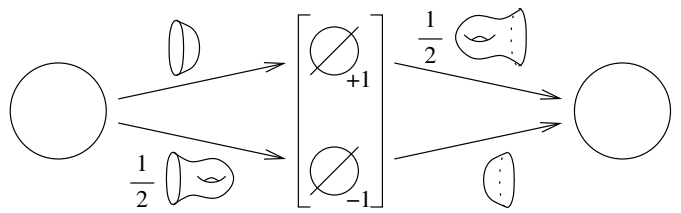
The case of tangles:



What else is it good for? It's local!

1. A localized relation with Kauffman's bracket.
2. Easily generalizes to surfaces, virtuals, etc.
3. Better understanding of functoriality.
4. Removing G and replacing NC with 4Tu yields a more general theory!

Complex simplification:



The Reduction Lemma. If ϕ is an isomorphism then the complex

$$[\bullet] \xrightarrow{\begin{pmatrix} \alpha \\ \beta \end{pmatrix}} \left[\begin{array}{c} \cdot \\ \cdot \end{array} \right] \xrightarrow{\begin{pmatrix} \phi & \delta \\ \gamma & \epsilon \end{pmatrix}} \left[\begin{array}{c} \cdot \\ \cdot \end{array} \right] \xrightarrow{(\mu \ \nu)} [\bullet]$$

is isomorphic to the (direct sum) complex

$$[\bullet] \xrightarrow{\begin{pmatrix} 0 \\ \beta \end{pmatrix}} \left[\begin{array}{c} \cdot \\ \cdot \end{array} \right] \xrightarrow{\begin{pmatrix} \phi & 0 \\ 0 & \epsilon - \gamma\phi^{-1}\delta \end{pmatrix}} \left[\begin{array}{c} \cdot \\ \cdot \end{array} \right] \xrightarrow{(0 \ \nu)} [\bullet]$$