

Stat 2211 Assignment 2

Due Thursday, March 5, beginning of class

Problem 1. Show that if $X_n \rightarrow X$ in probability and $|X_n| < Y$ for some Y with $\mathbf{E}|Y| < \infty$, then $X_n \rightarrow X$ in L^1 .

Problem 2. Show that if f is a function with $f(x) \rightarrow +\infty$ as $x \rightarrow \pm\infty$, and $\sup_i \mathbf{E}f(X_i) < \infty$, then the random variables $\{X_i\}$ are tight.

Problem 3. Start Pólya's urn with b black balls and w white balls. Show that the number of white balls over total number of balls converges almost surely to a limit X . Find the distribution of X . Prove your claims.

Problem 4. Let G be a recurrent graph. Show that all nonnegative harmonic functions G must be constant. (Hint: show that the harmonic function applied to the random walk gives a martingale).

Problem 5. 4.3.2 from Durrett. Give an example of a martingale so that $\sup_n |X_n| < \infty$ a.s. and $P(X_n = a \text{ i.o.}) = 1$ for $a = -1, 0, 1$.

Problem 6. Give an example of a martingale so that $X_n \rightarrow \infty$ a.s.

Problem 7. Wald's second equation. Let X_n be simple random walk on the integers, and let T be a stopping time with finite expectation. Show that $\mathbf{E}X_T^2 = \mathbf{E}T$.

Problem 8. Let X_n be a martingale with $X_0 = 0$ and $|\Delta X_n| \leq 1$ for all n . Show that there exists a constant c so that $\mathbf{E}X_n^4 \leq cn^2$ for all n .