

MAT 137
Tutorial #3— The definition of limit
October 1–2, 2018

Let f be a function. Let $a, L \in \mathbb{R}$. Assume that f is defined on some open interval around a , except maybe at a . As you know, the definition of the statement $\lim_{x \rightarrow a} f(x) = L$ is

For every $\varepsilon > 0$, there exists $\delta > 0$ such that $0 < |x - a| < \delta \implies |f(x) - L| < \varepsilon$.

Below is a list of eight other statements.

- a. For every $\varepsilon > 0$, there exists $\delta > 0$ such that $|x - a| < \delta \implies |f(x) - L| < \varepsilon$.
- b. For every $\varepsilon > 0$, there exists $\delta > 0$ such that $0 < |x - a| < \delta \implies 0 < |f(x) - L| < \varepsilon$.
- c. For every $\varepsilon \geq 0$, there exists $\delta > 0$ such that $0 < |x - a| < \delta \implies |f(x) - L| < \varepsilon$.
- d. For every $\varepsilon > 0$, there exists $\delta \geq 0$ such that $0 < |x - a| < \delta \implies |f(x) - L| < \varepsilon$.
- e. For every $\varepsilon > 0$, there exists $\delta > 0$ such that $0 < |x - a| < \delta \implies |f(x) - L| \leq \varepsilon$.
- f. For every $\delta > 0$, there exists $\varepsilon > 0$ such that $0 < |x - a| < \delta \implies |f(x) - L| < \varepsilon$.
- g. For every $\delta > 0$, there exists $\varepsilon > 0$ such that $0 < |x - a| < \varepsilon \implies |f(x) - L| < \delta$.
- h. There exists $\delta > 0$ such that for every $\varepsilon > 0$, $0 < |x - a| < \delta \implies |f(x) - L| < \varepsilon$.

Match each of the eight statements above to one of the following (there may be repeats):

- 1. Every function satisfies this statement.
- 2. There isn't any function which satisfies this statement.
- 3. This statement is equivalent to the definition of limit.
- 4. This statement means that $\lim_{x \rightarrow a} f(x) = L$ and that, in addition, $f(a) = L$.
- 5. This statement means that $\lim_{x \rightarrow a} f(x) = L$ and that, in addition, f does not take the value L anywhere on some interval centered at a , except maybe at a .
- 6. This statement is equivalent to saying that f must be constantly equal to L on an interval centered at a , except maybe at a .
- 7. This statement means that f is bounded on every interval centered at a .