

MAT 137
Tutorial #2– Proofs
September 24-25, 2018

Recall some proof techniques:

- To prove “ $\exists x \dots$ ”, say what x is.
- To prove “ $\forall x \dots$ ”, begin by fixing a generic x .
- To prove “ $P \implies Q$ ”, assume P is true and show that Q is true.

Write formal, rigorous proofs for these statements:

1. $\forall x \in [0, 1), \exists y \in [0, 1)$ such that $x < y$.
2. $\exists y \in \mathbb{R}$ such that $\forall x \in [0, 2], x^2 + 1 < y$.
3. If n is an odd integer, then $n^2 - 1$ is a multiple of 8.
4. For every positive number $x > 0$ and for every natural number $n \geq 2$,

$$(1 + x)^n > 1 + nx.$$

Hint: Use induction.