MAT 137
Tutorial #17– Sequences
February 25–26, 2019

1. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence. Write down the formal definition of the following concepts. You have already seen some of these in lecture.

   (a) The sequence is convergent.
   (b) The sequence is divergent.
   (c) The sequence is divergent to ∞.
   (d) The sequence is divergent to $-\infty$.
   (e) The sequence is increasing.
   (f) The sequence is decreasing.
   (g) The sequence is non-decreasing.
   (h) The sequence isn't decreasing.
   (i) The sequence is bounded above.
   (j) The sequence is not bounded above.
   (k) The sequence is bounded.

   Hints:
   Are all your variables introduced or properly quantified in Question 1a?
   Questions 1e, 1g and 1h all have different answers.

2. The following is a well-known result known as Stirling's formula:

$$\lim_{n\to\infty} \frac{n!}{n^n e^{-n}\sqrt{2\pi n}} = 1$$

   For this problem, you may assume we already know this formula is true. Use it to calculate the limits of the four sequences below.

   (a) $\lim_{n\to\infty} \frac{n!e^n}{n^{n+1/2}}$
   (b) $\lim_{n\to\infty} \frac{(2n)!}{e^{-2n}(2n)^{2n}\sqrt{n}}$
   (c) $\lim_{n\to\infty} \frac{(2n)!\sqrt{n}}{n!^2\, 4^n}$
   (d) $\lim_{n\to\infty} \frac{\sqrt[n]{n!}}{n}$