MAT 137
Tutorial #13– Areas
January 21–22, 2019

1. Calculate the area of the region between the curve $y = 1 + \sin(2x)$, the x-axis, the y-axis, and $x = \pi/2$.

2. Find the area of the region bounded by the graph $y = 4x - x^2$ and the x-axis.

3. Find the area of the region between the curves $y = 5 - x^2$ and $y = 3 - x$.

4. Calculate the area of the total region bounded by the curves $2y + 1 = 0$ and $\cos x + y = 0$, from $x = 0$ to $x = 2\pi$.

5. Find the *total* area of the region bounded by the curves $y = x^3 - 2x + 2$ and $y = x^2 + 2$.

6. We want to compute the area of the region between the line $y = x - 1$ and the parabola $y^2 = 2x + 6$ We have two methods to calculate this one. For both methods, start by finding the points of interception and sketching the region.

   (a) *Method 1.* Solve for y on the equation $y^2 = 2x + 6$. You will notice that there are two solutions. Look back at the graph. These correspond to the two branches of the parabola when we consider y as a function of x.

   Try to decompose your area as the sum of two areas that you can write as integrals. Be careful.

   (b) *Method 2.* Think of y as the variable and of x as a function of y.

7. Calculate the area of the region bounded by the curves $y^2 + 3y - x = 16$ and $|3y| - x = 0$.

8. Consider the collection of all parabolas with equations $y = (\cos a)(x^2 - \sin^2 a)$ where a is an arbitrary constant. Find the parabola such that the region between itself and the x-axis has the largest possible area.

Answers

1. $\pi/2 + 1$
2. $32/3$
3. $\dfrac{9}{2}$
4. $\pi/3 + 2\sqrt{3}$
5. $37/12$
6. 18
7. 192
8. ?