

**MAT 137**  
**Tutorial #10– L'Hôpital's Rule**  
**December 3-4, 2018**

1. Compute the following limits:

(a)  $\lim_{t \rightarrow 0} \frac{\tan(3t)}{\ln(1+2t)}$

(b)  $\lim_{h \rightarrow 2} \frac{h^3 - 5h^2 + 3h + 6}{h^3 - h^2 - 3h + 2}$

(c)  $\lim_{t \rightarrow 0} \frac{1 - \cos(3t)}{t \ln(1+t)}$

(d)  $\lim_{x \rightarrow \infty} (x^7 - 5x^3 + 2) e^{-x}$

(e)  $\lim_{x \rightarrow 1} \frac{(x-1) \sin x}{e^x \cos x}$

(f)  $\lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x}$

(g)  $\lim_{x \rightarrow \infty} x \tan \frac{3}{x}$

(h)  $\lim_{x \rightarrow 0} \left( \frac{1}{x \sin x} - \frac{1}{x \tan x} \right)$

(i)  $\lim_{x \rightarrow \infty} \frac{\sqrt{x^4 + 5x^3} - x^2}{x}$

(j)  $\lim_{x \rightarrow \infty} \frac{\sin x}{x}$

2. Compute the following limits:

(a)  $\lim_{x \rightarrow 0} (\cos x)^{1/x^2}$

(b)  $\lim_{x \rightarrow \infty} x^{\frac{\ln 2}{1+\ln x}}$

(c)  $\lim_{x \rightarrow 0^+} x^{\sqrt{x}}$

(d)  $\lim_{x \rightarrow 1} (2-x)^{\tan(\pi x/2)}$

## A historical question

3. The first appearance in print of L'Hôpital's Rule was in the book *Analyse des Infiniment Petits* published by the Marquis de l'Hôpital in 1696. This was the first calculus *textbook* ever published and the example that the Marquis used in that book to illustrate his rule was to find the limit of the function

$$y = \frac{\sqrt{2a^3x - x^4} - a\sqrt{aax}}{a - \sqrt[4]{ax^3}}$$

as  $x$  approaches  $a$ , where  $a > 0$  is a constant. (At that time it was common to write  $aa$  instead of  $a^2$ .) Solve this problem.<sup>1</sup>

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<sup>1</sup>Problem taken from *Stewart, James: Calculus Early Transcendental*, 7th Ed.