

**MAT 137**  
**Tutorial #1– Logic, quantifiers, and definitions**  
**September 17–18, 2018**

Let  $A$  be a subset of the real numbers. Francisco, Jeffrey, Qin, and Ivan tell us when they like it:

- Francisco likes  $A$  if and only if  $\exists a \in \mathbb{R}$  such that  $\forall x \in A, a \leq x$ .
- Jeffrey likes  $A$  if and only if  $\exists a \in A$  such that  $\forall x \in A, a \leq x$ .
- Qin likes  $A$  if and only if  $\forall x \in A, \exists a \in \mathbb{R}$  such that  $a < x$ .
- Ivan likes  $A$  if and only if  $\exists a \in A$  such that  $\forall x \in A, a < x$ .

Before you do anything else, try to understand with words which types of subsets each of the four people like. Then answer the following questions.

1. Is there any of them who likes every subset of the real numbers?
2. Is there any of them who does not like any subset of the real numbers?
3. Who likes the empty set?
4. Is the following statement true?

*For every  $A \subseteq \mathbb{R}$ , if Francisco likes  $A$  then Jeffrey likes  $A$ .*

5. Is the following statement true?

*For every  $A \subseteq \mathbb{R}$ , if Jeffrey likes  $A$  then Francisco likes  $A$ .*

6. The following proof is wrong. Explain why it is wrong. Then rewrite the proof so that it is correct.

**Theorem:** If  $A_1$  and  $A_2$  are two sets of real numbers that Francisco like, then Francisco also likes  $A_1 \cup A_2$ .

**Proof:**

- Since Francisco likes  $A_1$ , we know that for all  $x \in A_1, a \leq x$ . (I)
- Since Francisco likes  $A_2$ , we know that for all  $x \in A_2, a \leq x$ . (II)
- We need to prove that for all  $x \in A_1 \cup A_2, a \leq x$ .

Let  $x \in A_1 \cup A_2$ .

- Either  $x \in A_1$  — in this case  $a \leq x$  by (I).
- Or  $x \in A_2$  — in this case  $a \leq x$  by (II).

In both cases  $a \leq x$  so we are done.  $\square$