MAT 137Y: Calculus with proofs
Assignment 9 - Comments and common errors

Fatal errors

Most convergence tests only work in one direction. You can't use them backwards. If you try to, your answer is completely flawed and you won't get any marks.

- IF $\lim_{n\to\infty} a_n = 0$, we CANNOT conclude that $\sum_n^\infty a_n$ is convergent.
- IF $\sum_n^\infty a_n$ is convergent, we CANNOT conclude that $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$.

Q1a

- Do not confuse a series with a sequence.
  Do not confuse convergence of a sequence with convergence of a series.
  Do not confuse the sum of a series with the limit of a sequence.
- You may only use limit laws if you know all the involved limit exists. For example, you may only write
  $$\lim_{n\to\infty} (E_n + O_n) = \lim_{n\to\infty} E_n + \lim_{n\to\infty} O_n$$
  if you know that both $\lim_{n\to\infty} E_n$ and $\lim_{n\to\infty} O_n$ exist.

Q1b

- Just because you used both hypotheses in the proof, it does not mean that the theorem is false without them. Perhaps your proof does not work if we omit one of the hypotheses, but there might be a different, simpler proof that does work. The only way to prove that the theorem is false when we omit the hypotheses is with examples.

  Moreover, saying that something is possible (e.g.: "it is possible for $\lim_{n\to\infty} S_{2n}$ to exist while $\lim_{n\to\infty} a_n \neq 0$") does not actually prove it is possible. Once again, to prove it is indeed possible you need to provide an example.

Q2a

- $\sum_n^\infty \frac{1}{n^n}$ is NOT a p-series, and is NOT a geometric series.

Q2bcd

- The Big Theorem gives us comparison as $n \to \infty$, such as $\ln n << n$.

  We CANNOT use Big Theorem to conclude, for example that $\ln a_n << a_n$ because $a_n \not\to 0$

Q2e

- If every test you tried is inconclusive, it does not mean that "we cannot draw any conclusion". Perhaps there is a different argument you have not thought of. The only way to prove that "we cannot draw any conclusion" is with two examples.
- It is NOT true that $\sqrt{x} \leq x$ for all $x \geq 0$. It is only true for $x \geq 1$.
- The absolute convergence test does not work backwards.

  IF $\sum_n \sqrt{a_n}$ is divergent, we CANNOT conclude that $\sum_n (-1)^n \sqrt{a_n}$ is also divergent.

Q2f

- We CANNOT assume that for all $x \in [-1, 1]$, $f(-1) \leq f(x) \leq f(1)$.
  Not every function is increasing!
- We CANNOT use BCT on a series that has both positive and negative terms. If you try to do so, your answer is entirely wrong.