MAT 137Y: Calculus with proofs
Assignment 6 - Comments and common errors

Global errors

- **The lower integral of a function is NOT one of the lower sums.** We define the lower integral as a supremum, rather than as a maximum, for a good reason.
- The notation
$$\{ \text{ for all } P \mid L_P(f)\}$$
means nothing. Please review set-building notation (Unit 1). Rather, you can write (for example)
$$\{L_P(f) \mid P \text{ is a partition of } [a,b]\}$$
- The notation
$$\{L_P(f) \mid \forall P \in [a,b]\}$$
is wrong for two reasons. First, the "for all" should not be there. Second, P is not an element of $[a,b]$.

Q1b

- Infimum is not the same as minimum. Not every bounded function has a minimum. That is why we use infimum instead!
- You probably gave names to the infima of f, g, and h on each subinterval. Whatever notation you choose to use, explain it! We can't read your mind.

Q3

- This was similar to an "ε-δ proof", like in Unit 2. Remember everything you learned about proof structure, introducing your variables in order, paying attention to what depends on what...
- Using Question 3 you can conclude that there exist partitions P_1 and P_2 such that
$$\begin{aligned} \underline{I_a^b}(f) - \varepsilon/2 &< L_{P_1}(f) \\ \underline{I_a^b}(g) - \varepsilon/2 &< L_{P_2}(g) \end{aligned}$$
but **there is no reason why P_1 and P_2 should be the same.** If you made this error your proof is entirely wrong and your grade will be 0.
- If you take a partition P finer than P_1 and P_2, how do you know that such a partition exists? You could instead take explicitly $P = P_1 \cup P_2$, for example.

Q4

- Notice that in the statement

$$\forall \varepsilon > 0, \ \ \exists \text{ partition } P \ \text{ s.t. } \ \underline{I}_a^b(f) + \underline{I}_a^b(g) - \varepsilon < L_P(f) + L_P(g)$$

  the partition P depends on ε.

  For example, if you begin your proof by going from

$$\underline{I}_a^b(f) + \underline{I}_a^b(g) - \varepsilon < L_P(f) + L_P(g)$$

  to

$$\underline{I}_a^b(f) + \underline{I}_a^b(g) \leq L_P(f) + L_P(g)$$

  then your proof is entirely wrong. To do this kind of reasoning you have to pretend that the same partition P works for all values of $\varepsilon > 0$.

  Many of you made the same error in other contexts. Please review your quantifiers (Unit 1).

  If you made this error, your grade on this question will be 0. Your whole argument is wrong.

- In Q3, ε and P are quantified. They do not have any intrinsic meaning. If you use them in your proof, you need to introduce them. Review how to work with quantifiers (Units 1 and 2).

- If you want to use properties involving $\lim\limits_{||P|| \to 0}$, such as the limit laws, you will need to state and justify them. These are entirely new types of limits and do not behave the same way as limits of functions as in Unit 2.

Q7

- The Squeeze Theorem has nothing to do with this question. Review the statement of the Squeeze Theorem: it is about inequalities with functions, not with numbers.