MAT 137Y: Calculus with proofs
Assignment 4 - Comments and common errors

Q1a

- f is a function; $f(x)$ is a number.

Q1b

- The negation of a conditional is never a conditional. If in doubt, review Video 1.9.
- To verify that $f \circ g = f \circ h$, you need to verify that *for every* $x \in \mathbb{R}$, $f(g(x)) = f(h(x))$. It is not enough to verify it for one single value of x.
- Remember the difference between a quantified variable (or function) and a fixed one.
  - If you write "Let g be a function" you are a fixing an arbitrary function. You cannot later choose a specific g.
  - If you want to define the function g, write

    "For every x, $g(x) = \ldots$"

    rather than

    "Let $x \in \mathbb{R}$. $g(x) = \ldots$"

    Otherwise you are defining g at one single value of x.

Q2

- You cannot choose a function f that depends on c. We are asking you to

  "construct a function f such that for every c ..."

  not

  "for every c a function f such that ..."

  If your function f depends on c, your grade is automatically 0. This is a serious error. You need to review Unit 1.
- Notice that you need to find a sequence of points x_n such that $f(x_n)$ and $f'(x_n)$ gets arbitrarily large. It is not enough that $f'(x_n)$ increases: it needs to get arbitrarily large.
- Similarly, you need to notice why the restriction of f to an open interval containing the point x_n is one-to-one. Otherwise there is no quasi-inverse.

Q3a

- In order to "take ln out of the limit", you need to notice that ln is a continuous function. (That is Theorem 3 in Video 2.16, which we invited you to use.)
- If you use the theorem labelled as "False theorem" in Video 2.16, you made a TA very sad. You also did not get any points.
- There is no reasonable way to use "Theorem 2" from Video 2.16. You would need to prove that for every x close to 0 but not 0, $(1+x)^{1/x} \neq e$, and there is no easy way to prove such a thing.

Q3b

- Many of you used a change of variables without justifying it. We had already hinted you at Video 2.16 in Q3a. You needed to use Theorem 2 from Video 2.16 for Q3b.

Q4c

- Concluding that the theorem is valid "as is" when $x > 0$ is only half the problem. You still need to find the correct statement when $x < 0$.
- Even if you notice that the theorem is true when $x > 0$, you still need to justify why. The proof that was presented was flawed. You cannot simply copy the same proof. Why does the proof work for $x > 0$ but not for $x < 0$?