MAT 137Y: Calculus with proofs
Assignment 1 - Comments and common errors

Some of you have probably received a grade much lower than expected. Do not despair yet!

This is normal. You are learning to write proofs for the first time. This is difficult. It takes time to master and to get used to rigorous standards. If you feel that you are making progress, if things are starting to make sense, if you read the sample solutions and comments and you understand where you went wrong, don't be disheartened.

Q0

The question asked to copy the statement and sign it. We were generous in what counts as a signature, but if you did not sign it, then you get no point. Please get in the habit of reading instructions.

Q1a

- If you are trying to prove "$\forall x \in \Omega_f^g,\ \exists y \in \Omega_g^f,$ such that $x < y$" you cannot choose specific values of x. You must prove it *for all* x.
- Use correct notation. The following is correct:

$$\Omega_{\text{Walt}}^{\text{Tor}} = \{x \in \mathbb{R} \mid \exists n \in \mathbb{Z} \text{ s.t. } (2n-1)\pi < x < 2n\pi\}$$

  All of the following are wrong:

$$\begin{aligned}
\Omega_{\text{Walt}}^{\text{Tor}} &= \{x \in \mathbb{R} \mid \forall n \in \mathbb{Z},\ (2n-1)\pi < x < 2n\pi\} \\
\Omega_{\text{Walt}}^{\text{Tor}} &= \{x \in \mathbb{R} \mid n \in \mathbb{Z},\ (2n-1)\pi < x < 2n\pi\} \\
\Omega_{\text{Walt}}^{\text{Tor}} &= \{x \in \mathbb{R} \mid (2n-1)\pi < x < 2n\pi,\ n \in \mathbb{Z}\} \\
\Omega_{\text{Walt}}^{\text{Tor}} &= \{(2n-1)\pi < x < 2n\pi, n \in \mathbb{Z}\} \\
\Omega_{\text{Walt}}^{\text{Tor}} &= ((2n-1)\pi, 2n\pi)
\end{aligned}$$

  If this confuses you, review Question 7 on the Unit 1 Practice Problems.
- Use quantifiers correctly. The statement

$$x \in (2n\pi, (2n+1)\pi)\ \forall n \in \mathbb{Z}$$

  is never true.

Q1b

- If you did not get this question right, review Video 1.8.

Q1c

- $\emptyset$ is not the same as $\{\emptyset\}$. The set $\{\emptyset\}$ is not empty because it contains one element: the empty set.
- The question asks you to find all the functions f that satisfy that "f loves f". Some of you had the right idea but never answered the actual question.
- If you did not get this question right, review Video 1.6.

Q2a

- A graph can help understand what is going on, and it can be help your reader understand the proof, but you cannot reduce your proof simply to "Look at the graph".
- This question only allows for functions with domain $\mathbb{R}$.
- Some of you used approximate values (making the argument wrong) and even worse, without explaining where they came from.

Q2b

- You are trying to prove that

  "For every function f, there exists a function g, such that for every $t \in \mathbb{R}$ ..."

  You need to construct a function g, depending on f, **BUT NOT DEPENDING ON** t.

  If this confuses you, review Video 1.4.
- Finding ONE function f and ONE function g that fail that "$\forall t \in \mathbb{R}$, g loves f_t" is not enough to disprove the statement. There could be a different function g that works for that function f.
- You cannot use derivatives in this question. You need to prove it for all functions, and f may not be differentiable.
- Infinity is not a number. You cannot define $g(x) = \infty$.

Q3

- If you are planning to use the notation "$P(n)$" or "S_n", introduce it first. Otherwise, your reader does not know what you are talking about.
- Some of you are calling something "$P(n)$" or "S_n". What is it?
  - If you declare "$P(n) = 5^{2n} + 11$", then $P(n)$ is not a statement. It does not make sense to say "Assume $P(n)$" or "$P(n)$ is true". However, in this case, you can write "$P(n)$ is a multiple of 12".

- If you declare "$P(n)$" to be the statement "$5^{2n} + 11$ is a multiple of 12", then it does make sense to write "Assume $P(n)$" or "$P(n)$ is true".

- The induction step works on one value of n at a time. In you induction step, **YOU FIX ONE POSITIVE INTEGER** n, you assume that $5^{2n} + 11$ is a multiple of 12, and you prove that $5^{2(n+1)} + 11$ is also a multiple of 12 for that specific value of n. If, instead, you write

  "Assume $\forall n$, $5^{2n} + 11$ is a multiple of 12",

  then your proof is wrong. You are assuming too much (you are assuming what you want to prove!)

- A proof by induction has 2 steps, not 3 steps.